615–Maths Methods in Theoretical Physics

Problem Sheet 1

(**1**) Suppose

$$\frac{dy}{dx} = g(y/x)$$

where g is an arbitrary function of the ratio y/x. Show that the susbtitution $u = y/x$ leads to a separable equation for u as a function of x, and present the general solution. (*i.e.* reduce to quadratures.)

(**2**) Determine which of the following are exact differentials, and which are not:

$$(a) \quad \omega = ydx + xdy\,, \qquad (b) \quad \omega = ydx - xdy\,,$$
$$(c) \quad \omega = \sin y\, dx + \cos y\, dy\,, \qquad (d) \quad (x+y)dx + \tan x\, dy$$

In each case that is not exact, find an integrating factor that renders it exact. (Hint: It is sometimes useful to try integrating factors that are functions of x only or y only.)

(**3**) By finding an appropriate integrating factor, solve

$$\frac{dy}{dx} = -\frac{2x^2 + y^2 + x}{xy}$$

(**4**) Construct the two independent solutions of the the second-order ODE $y'' + y = 0$ by applying Frobenius's method of seeking series solutions of the form

$$y(x) = \sum_{n \geq 0} a_n\, x^n$$

(Obtain the recursion relation for the coefficients a_n, and then solve it to obtain the two solutions as infinite series, with explicit expressions for the coefficients.)

Due Tuesday 13th Septembern in class