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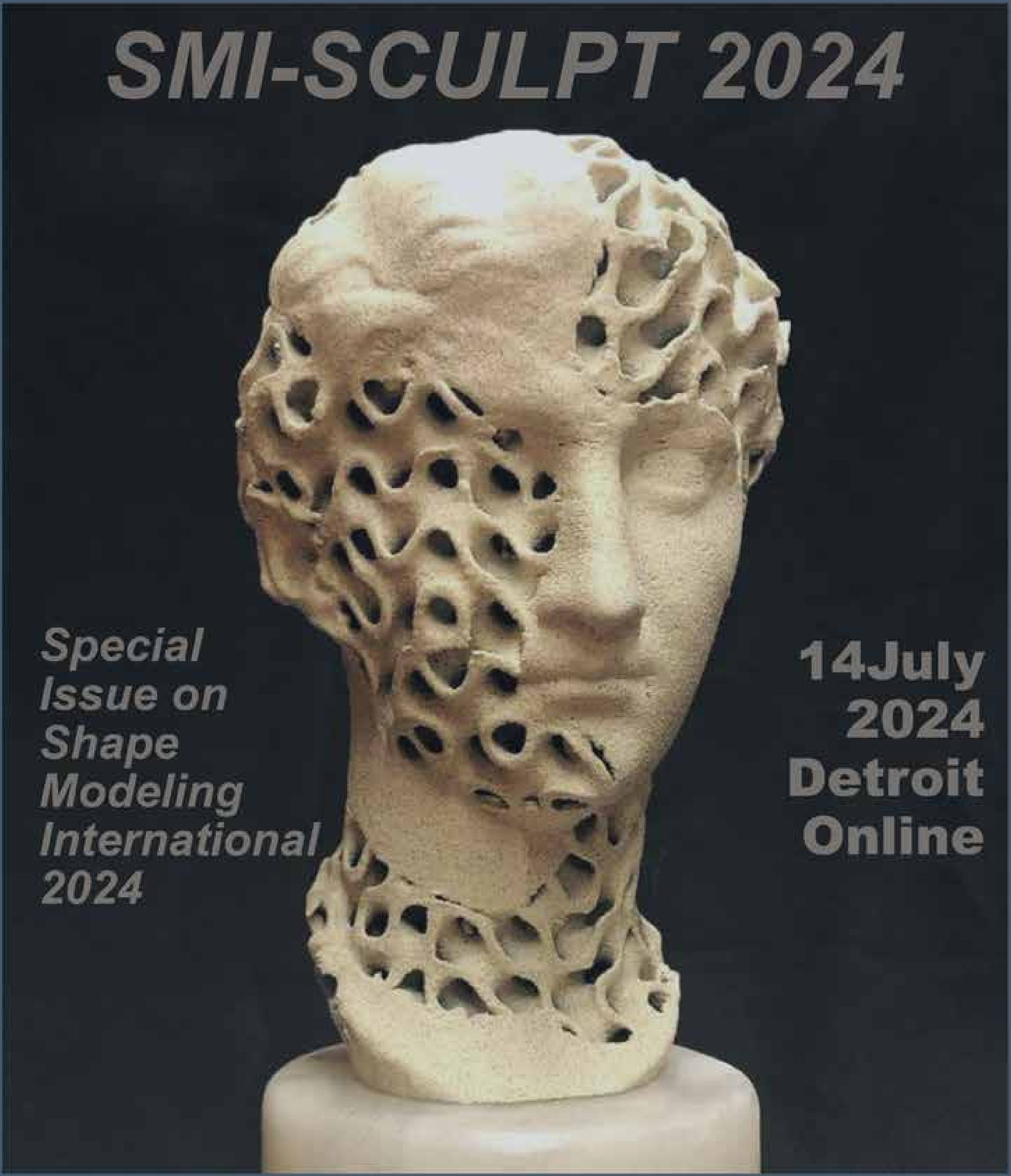

SMI-SCULPT 2024

Special
Issue on
Shape
Modeling
International
2024

14July
2024
Detroit
Online


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HYPERSEEING
Special Issue on SMI-SCULPT 2024

Shape Modeling International 2024
Shape Creation Using Layouts, Programs, & Technology (SCULPT) Event

Twenty third Interdisciplinary Conference of the International Society of the Arts, Mathematics, and Architecture

Online Event
July 14, 2024

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Preface

History of Shape Creation Using Layouts, Programs, & Technology (SCULPT) Event

The SCULPT event began as an experimental extension of the Shape Modeling International (SMI) conference in 2012. Originally titled the Fabrication and Sculpting Event (FASE), it debuted at SMI 2012 and returned again at SMI 2013. Both events were met with enthusiastic responses to the FASE papers and presentations. Although there was no FASE event at SMI 2014, the success of the initial editions led us to continue FASE as part of the SMI conference annually from 2015 to 2021.

In 2013, Nat Friedman—Chair of the International Society of the Arts, Mathematics, and Architecture (ISAMA)—approached me with the idea of organizing the event as an annual ISAMA conference. I proposed the suggestion to the SMI Steering Committee, which unanimously supported the idea. As a result, the event now also serves as the Twentieth Interdisciplinary Conference of ISAMA.

ISAMA itself has a rich and meaningful history. The first Art and Mathematics Conference (AM 92) was organized by Nat Friedman at SUNY Albany in June 1992. This was followed by annual events AM 93 through AM 97 in Albany and AM 98 at the University of California, Berkeley, co-organized with Carlo Séquin. ISAMA was officially founded by Nat Friedman in 1998, and the ISAMA publication Hyperseeing was co-founded by him and me in 2006.

Over the years, the art/math movement has grown significantly, with the emergence of new conferences and organizations. Notably, the Bridges conference, launched by Reza Sarhangi in 1998, has become highly successful, with excellent proceedings and international recognition. The importance of the art/math movement is also highlighted by the prominent art/math exhibit at the annual Joint Mathematics Meetings of the American Mathematical Society and the Mathematical Association of America, organized by Robert Fathauer.

In 2022, we renamed the event SCULPT to better convey its mission. SCULPT stands for Shape Creation Using Layouts, Programs, & Technology. Unlike other math/art conferences, SCULPT is uniquely focused on the physical realization of 3D shapes. We primarily invite submissions from practitioners such as sculptors and architects to describe their creative methods. These contributions, along with the discussions they spark, are expected to raise new questions and challenges for theoretical research in shape modeling.

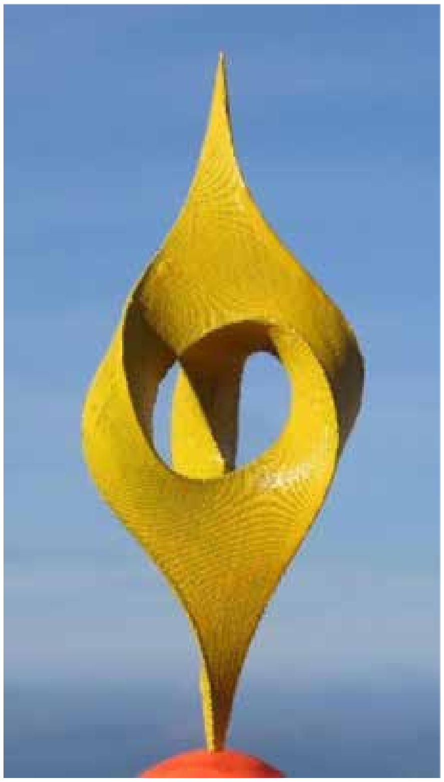
In 2023, we introduced a new *Show and Tell* event as part of SCULPT, held virtually via Zoom. In this format, participating sculptors briefly present their work in 5-minute segments. Each session has featured more than 15 artists, showcasing a wide range of creative processes and techniques. This informal yet dynamic platform has helped foster greater engagement and community among artists, designers, and researchers. Beginning with this edition, we now include the showcased works as part of the official proceedings, helping to document and share the contributions of our creative participants more broadly.

Ergun Akleman

Editor, Hyperseeing

Preface

Shape Creation Using Layouts, Programs, & Technology (SCULPT) 2023



There are at least two aspects to shape modeling: theoretical and practical. The mathematical and theoretical aspects of shape modeling have traditionally been supported by the SMI conference. With the Fabrication and Sculpting Event (FASE) our goal is to include more hands-on, application-oriented ways by designers and sculptors who construct sophisticated real-world objects.

FASE has its own program committee, and the accepted papers are published in Hyperseeing. With FASE, we hope to attract practitioners who might usually be less inclined to write papers containing formal algorithms or mathematical proofs, but who nevertheless have important things to say that are of interest to the shape modeling community and who also might provide visually stimulating material.

For this year's Fabrication and Sculpting Event, we solicited papers that pose new questions and motivate further research in designing, fabrication and sculpting. Topics should be useful, for example, in the following areas: Fabrication of digital models, Advanced manufacturing techniques such as additive manufacturing, laser cutting or CNC milling, Interactive or procedural design of manufacturable shapes, Interconnections of complex modeling and fabrication processes, visually stimulating fabrication techniques or printed structures.

Thus, the scope of FASE is the intersection of shape modeling and fabrication methods/algorithms, and papers may focus on both the digital/theoretical and the physical domain or just one of these domains – as long as the connection to the other domain is clear. It is not a requirement that the techniques presented in the paper involve computation as such, but they need to have a clear algorithmic or mathematical element.

We received six submissions this year and three of them were accepted as regular papers. The three accepted papers span a wide range of topics and views on the fabrication process of various artistically interesting artifacts. We wish to thank the authors and the reviewers for their participation in the SMI/ISAMA 2024 SCULPT Event. We hope that new ideas and partnerships will emerge from the FASE papers that can offer a glimpse into a much larger territory and the event can enrich interdisciplinary research in Shape Modeling. We hope that the attendees of SMI 2024 will enjoy this event of the conference.

Oleg Fryazinov, and Carlo Séquin

SCULPT Papers chairs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Tengstrand’s “3-2-1”-Sculpture and New Derivatives

Carlo H. Séquin
CS Division, University of California, Berkeley, CA, USA; sequin@berkeley.edu

Abstract

This is an analysis of the geometry of the “3-2-1”-sculpture by Tord Tengstrand, presented at the Bridges 2020 Art Exhibition. Derivative shapes are then obtained by changing some of the geometrical parameters. In one extension, the 3-fold rotational symmetry of the original sculpture is changed to values ranging from 2 to 6. In another extension, the complexity of the iterated edge-curve is enhanced to make more passes around the central void. In addition, one half of some of those derivative geometries are used as a modular building block for the more complex polyhedral frames structures based on the Platonic solids.

1. Introduction

In the Bridges 2020 Art Exhibition, Tord Tengstrand [7] presented an intriguing sculpture (Fig.1a) called ”3-2-1”, since it had 3 curved edges, 2 pointy vertices, but only a single, higly curved, multi-branch “face” that connects to itself across the three “loopy” edges. I don’t recall whether I saw it in 2020; but recently I came across an image of it in the Bridges archives. From a single image it is difficult to understand this shape. A lively email exchange helped Tord and me to understand this shape and its topological parameters.

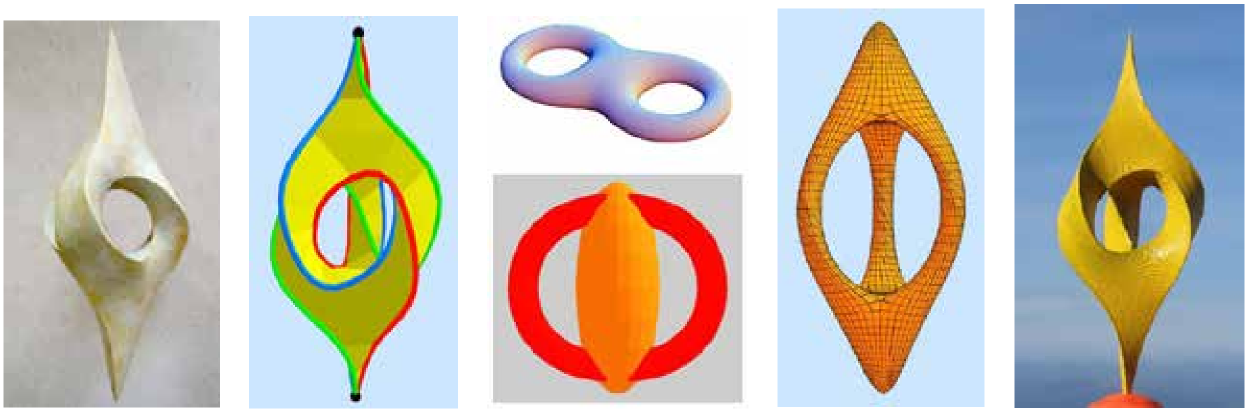
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Figure 1: *: (a) Tengstrand’s “3-2-1”-Sculpture. (b) CAD model showing the 2 vertices and the 3 edges; (c,d) handlebodies of genus 2. (e) 3D-print, painted yellow.*

Tord’s “3-2-1”-sculpture is a handlebody of genus 2 with 3-fold rotational symmetry around an axis that passes through the two (black) vertices in Figure 1b. Even though this model seems to have three “tunnels,” only two of its three solid prismatic branches need to be cut to turn it into a genus-0 object. Thus, the genus of this handlebody is 2; and it is topologically equivalent to a 2-hole torus, or to a spherical blob with two handles attached to it (Fig.1c). Embedded in a handlebody with 3-fold rotational symmetry (Fig.1d) are three identical, loopy edges with a dihedral angle of about 60 degrees (shown in red, green, and blue) that run from one vertex to the other one (Fig.1b). Each edge makes a “down-up-down” zig-zag motion that passes the central void three times. I call this a “3-pass edge.”

Between neighboring edges there are three “N”-shaped ribbons (shown in yellow, orange, and magenta in Figs.2a,b & 3a), which also make 3-pass, up-down, zig-zag moves that start at one 3-way junction and end at the other one. These ribbons border one another across the three loopy edges, but they are also smoothly connected to one another in the two junction areas. The resulting topology of the resulting overall “face” is shown in Figure 3d. It can also be seen as a hollow sphere with three slits in it (Fig.3e).

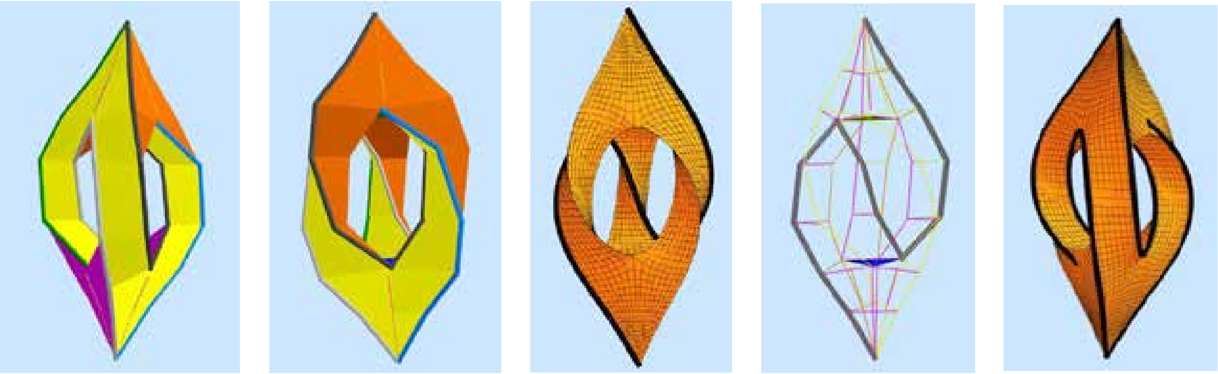
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Figure 2: *The 3 edges in the "3-2-1"-Sculpture: (a,b) Front and back views of a polyhedral model with sharp edges highlighted. (c,d) A single edge on the smoothed model, and in isolation. (e) Edges segments needed to define the sharp edges around a single ribbon component.*

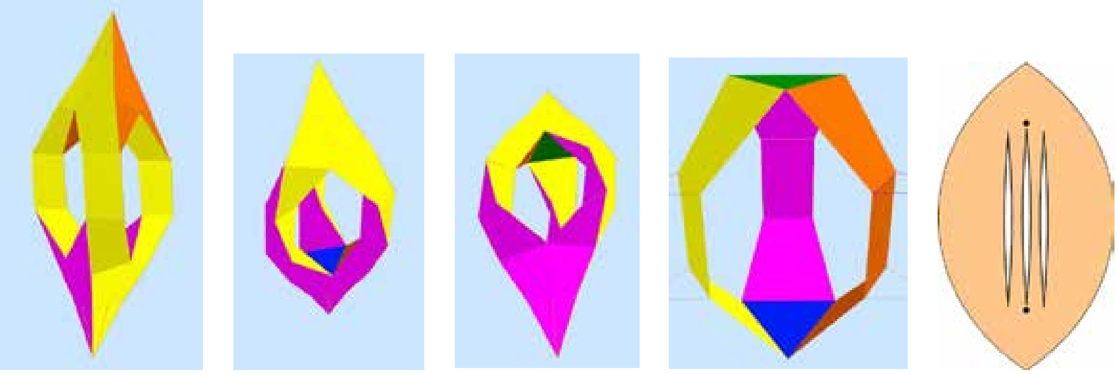
(a) (b) (c) (d) (e)

Figure 3: *The face of the "3-2-1"-sculpture: (a) The zig-zag motion of one ribbon-like component; (b,c) lower and upper 3-way junction areas; (d,e) simple topological models of the whole face.*

Modeling Issues

To make a clean CAD model of this sculpture, our home-brewed JIPCAD environment (Joint Interactive & Procedural Computer-Aided Design) [2],[3],[4] is well suited. It allows the user to specify some geometry, such as the loopy edges, in a procedural manner, e.g., as cubic B-splines. Surface facets between these edges can then be added interactively through the graphical user interface. The symmetry of these shapes can easily be exploited by defining just one loopy edge and then instantiating that edge three times with rotations of 120 degrees between them. Similarly, the D_3 symmetry of this shapes requires that only 1/6 of the overall surface must be constructed explicitly. The bottom half of the model can be obtained by flipping the top half through 180 degrees around the x-axis. This is how I created my first CAD model.

I started my design based on one edge-curve because this was a key feature in the "3-2-1"-sculpture. I modeled half of the edge-curve with a B-spline, displaying it with about 12 discrete linear segments. Then, I properly placed three copies of it and turned on "*vertex-selection*" in our CAD tool. This displays all vertices as selectable entities that can be used to define facets between them (Fig.4b). By selecting two pairs of vertices on two neighboring edge-curves I could form a small quad face. Near the two tips of the sculpture, it was easy to construct good facets; sometimes I used just a triangle (Fig.4c). In the middle of the sculpture, it is more difficult! In some places, the vertices do not line up in an obvious way. Nevertheless, I found a valid solution (Fig.4d) that forms a closed surface with no open border segments, and which resulted in a good polyhedral model (Fig.4e).

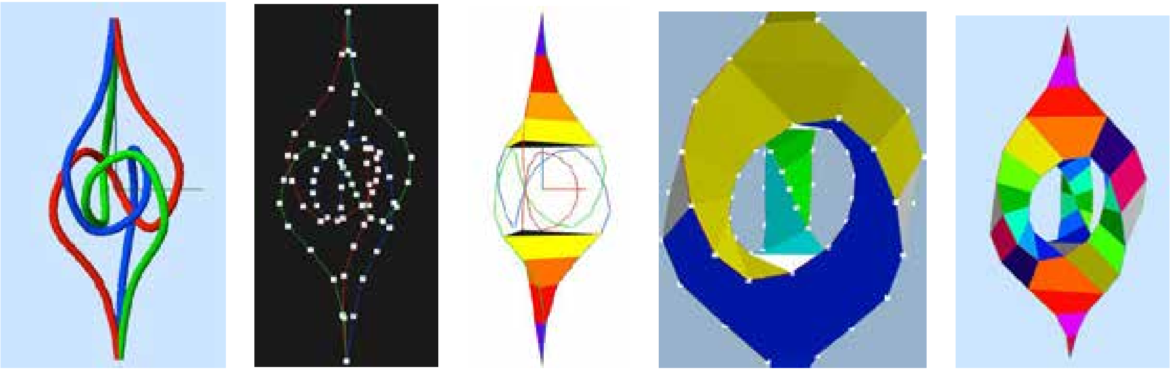
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Figure 4: *Modeling based on edges: (a) Desired edge configuration. (b) Displaying selectable vertices. (c) Modeling the 2 pyramids; (d) constructing the central parts; (e) resulting polyhedral model.*

However, even if I could properly close the surface, the three resulting prismatic branches my not exhibit a nice cross-section in the shape of an equilateral triangle. So, I switched to a different modeling approach. I found it preferable to model the three solid branches directly with 3-sided prismatic beams (Fig.5a), which are then connected to the two 3-sided pyramids that support the top and bottom vertices (Fig.5b). By recoloring the facets, I can identify the three ribbon-like surface areas (yellow, orange, magenta) (Fig.5c), and I can find out what edges have to be labeled as "sharp." To obtain the desired smooth surface (Fig.5d), the enhanced polyhedral model is subjected to three levels of Catmull-Clark subdivision [1] in which the sculptural sharp edges are prevented from being rounded in the subdivision process. This results in nice, smoothly shaped face strips, even when the starting polyhedral model is rather coarse. This model can then be converted into an STL-file and sent to a low-end 3D-printer [8]. The first model was printed in a dark filament, and its intriguing, curved, ribbon-like faces did not photograph well; therefore, I painted it in a bright yellow color (Fig.1e).

Actually, defining the sharp edges in these polyhedral models is somewhat challenging. Again, I want to exploit the symmetry of these objects, and I don't want to have to specify every single sharp edge in a hierarchically flat, polyhedral model. I want to define just one "ribbon country" (yellow in Fig.5c) and then use it two more times (shown in orange and magenta). Therefore, I must make sure that all the edges of the prototype ribbon face are marked as *sharp*. The four polylines marked in green, silver, black, and blue in Figures 2(a,b) achieve that goal. Figure 5d shows the result of smoothing the polyhedral model with three levels of CC-subdivision, and Figure 5e depicts the resulting shape with a proper set of *sharp* edges in the right places.

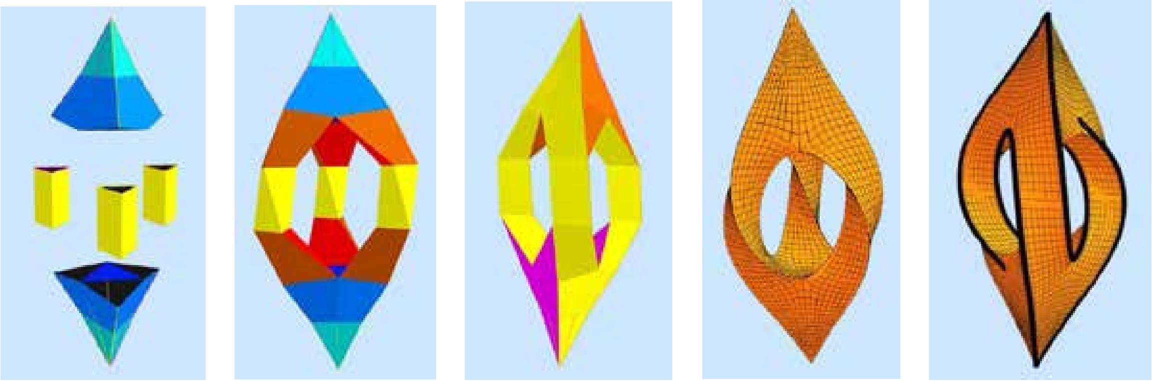
(a) (b) (c) (d) (e)

Figure 5: *Polyhedral modeling: (a) Placing pyramids and prisms; (b) components connected; (c) displaying the 3 ribbons countries; (d) after subdivision; (e) "sharp" edges enhanced.*

Derivative Sculptural Models with 3-Pass-Edges

In my paper for the 2024 Bridges Math-Art conference [5], I present several "derivative" models based on Tord's inspiring sculpture. In that first exploration, I increased the genus of the model by creating some "*E*-2-1"-geometries. In these models, all *E* edges start at the top vertex, move downwards past the central void, circle partially around two of the tunnels and then move downwards again to stop at the bottom vertex.

As the first derivative model, I created a "4-2-1"-handlebody. Its set of 4 loopy edges is shown in Figure 6a. However, I directly built a polyhedral model based on two 4-sided pyramids and four twisted, 3-sided prismatic legs (Fig.6b). Only one eighth of the new complex face of this sculpture needed to be constructed. The resulting polyhedral model (Fig.6c) can then be smoothed by Catmull-Clark-subdivision and turned into an STL-file that can be used to make a 3D-print (Fig.6d).

Figure 6: *Increasing the rotational symmetry: (a) The 4 edges of a "4-2-1"-sculpture; (b) placing pyramids and prisms; (c) connecting the components; (d) 3D-print. (e) CAD model of a "5-2-1"-sculpture; (f) a 3D-print.*

The same approach also allowed me to make a "5-2-1"-sculpture (Figs.6e,f). It was not difficult to make these extended derivative shapes, since the behavior of all the edges in all the sculptures is always the same: Each edge starts at the top vertex, travels half-way down the sculpture, loops partially, counter-clockwise around one tunnel/window, travels along the inside of one of the prismatic branches, then partially loops clockwise around the adjacent tunnel/window, and then heads for the bottom vertex. To make the extensions shown in Figure 6, I just had to repeat this edge-behavior *E* times around the vertical rotational symmetry axis. The resulting surfaces can be understood as follows: There are *E* ribbon-like parts that run in a zig-zag manner from the upper *E*-way junction to the lower one. Each such ribbon starts at the internal junction patch and uses a first 3-sided prismatic branch to move to the outside, where it passes into one of the *E* lower pyramid faces. From that pyramid face, the ribbon travels through the outside of a branch to a top pyramid face. Then, on the other leg of that pyramid face, the ribbon moves again to the inside, where it joins the lower *E*-way junction. Given that all these ribbons connect to both the upper and lower *E*-way junctions, it is clear that all surface pieces connect into a single smooth "*face*" bordered by the *E* sharp edges.

New Explorations Using 5-Pass Edges

In all of the above models, the branches had a triangular cross-section, and all the loopy edges made three passes past the central void. Now I want to accommodate more complicated edge-curves on branches that have more than three prismatic sides. All sculptures will still have just two vertices, and all the edge-curves have identical shapes.

My initial goal was to construct a structure, where each edge passes once through each one of the five prismatic branches and gradually winds its way around the whole structure, as it does in Tengstrand's original “3-2-1”-sculpture (Fig.7a). This new “wiggly” edge would make 5 passes past the central void as it works its way once around the bi-pyramid structure. In Figure 7b, a single (green) edge winds through the five prismatic branches in the desired manner. In Figure 7c, a second (blue) edge has been added, rotated around the symmetry axis by 72 degrees. However, with this arrangement of edges, I was not able to find a set of five ribbon-like faces that could define a proper handlebody of genus 4. Suspending ribbon-like surfaces between neighboring edges always resulted in twisted facets that intersected with facets from neighbouring ribbons. And, when a third (orange) edge is introduced (Fig.7d), the edges themselves start to intersect.

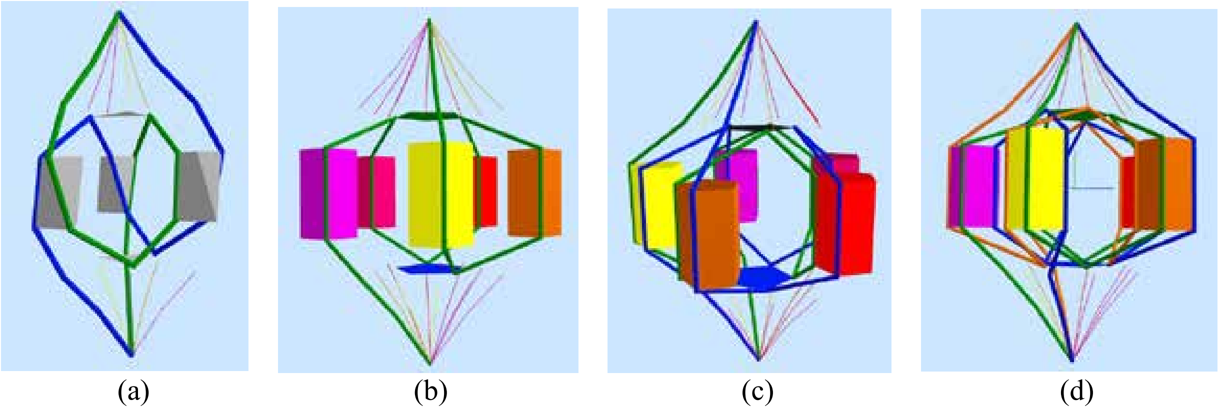
(a) (b) (c) (d)

Figure 7: *(a) Edge set-up of the original “3-2-1”-sculpture. (b) Similar edge setup for five pentagonal prism branches, with one edge highlighted; (c) a second such edge added; (d) three edges shown.*

On the other hand, I have been able to construct a good ribbon-face (yellow) that travels around just <u>two</u> adjacent tunnels. It starts at the upper 5-way junction, then passes through pyramid faces at both vertices (Fig.8a), and ends up at the lower 5-way junction. The ribbon travels three times along one prismatic branch, at the inside of which it runs adjacent to itself (Fig.8b); and it also runs once through both neighboring branches (Figs.8c). This then results in a 5-pass edge structure forming two loops (Figs.8d,e).

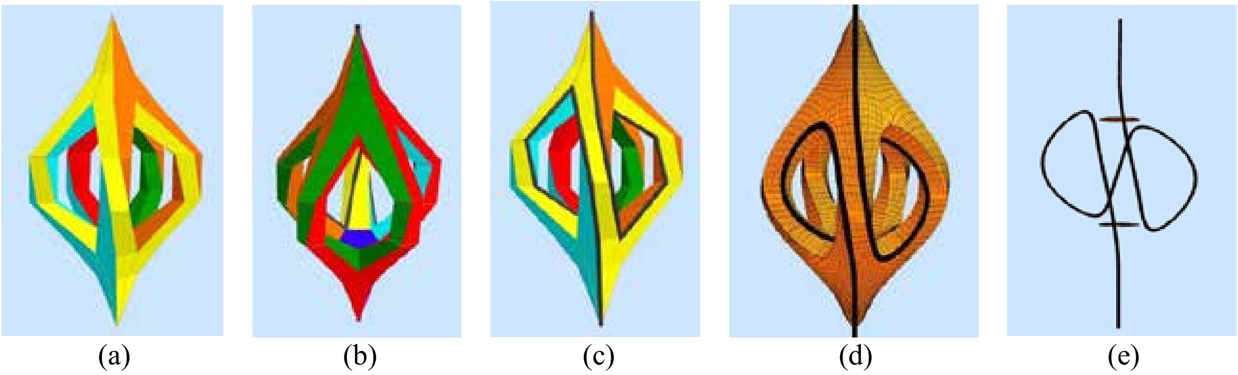
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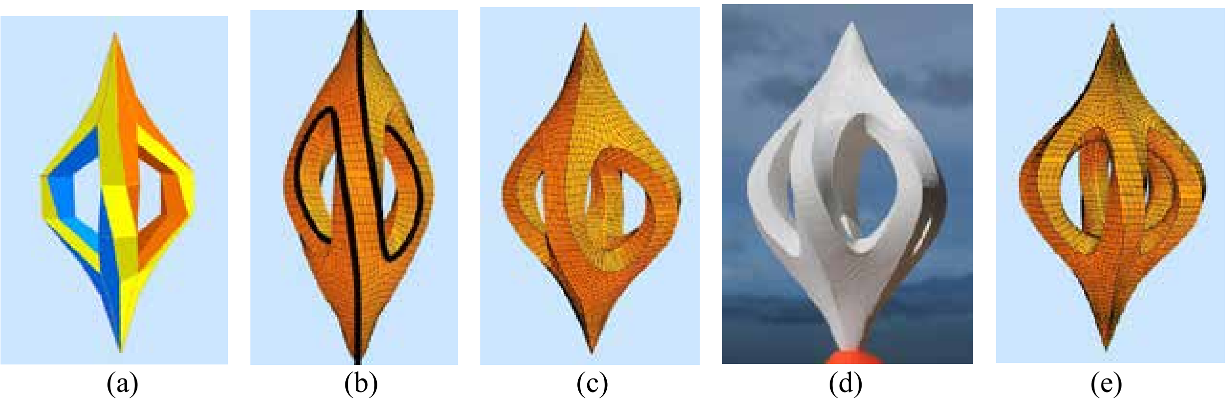
Figure 8: *A new type of a “5-2-1”-sculpture: (a) polyhedral model, (b) back-side view; (c) one edge highlighted. (d) after CC-subdivision; (e) extracted edge curve.*

This basic 5-pass, double-loop edge-shape can now be used in Tengstrand structures with a different number of prismatic branches, *B*. Figure 9 shows results for three, four, five, and six branches. In each case the 5-pass, double-loop edge-curves will be accommodated in three adjacent branches, as is also the case for the 3-pass edge-curve in the various enhanced Tengstrand sculptures shown in Figure 6. For *B*=*E*=3, this results in a new “3-2-1”-sculpture with a more complex edge shape (Figs.9a,b). There are always as many edges as there are branches in the handlebody (Figs.9c,d,e), and as long as all edges are of the 5-pass type, the branches are 5-sided prismtic beams.



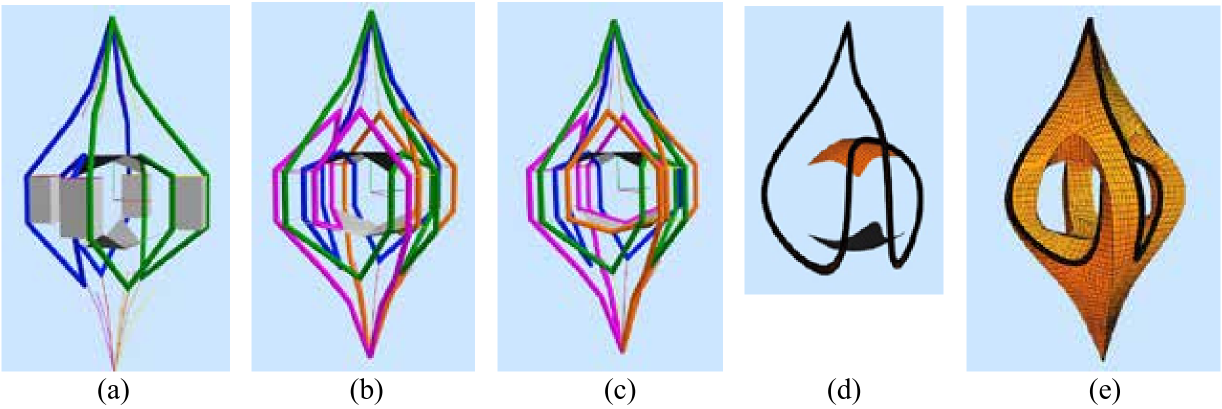
(a) (b) (c) (d) (e)

Figure 9: *Using 5-pass edges: (a) 3-branch "3-2-1"-polyhedral starting model; (b) model after 3-level CC-subdivision, with a single 5-pass edge highlighted; (c)"4-2-1"-CAD-model, (d) "5-2-1"-3D-print. (e)"6-2-1"-CAD-model.*

Derivative Tengstrand Structures with 4-Pass Edges

The reader may wonder why I jumped directly from the 3-pass edge structures that I explored in my Bridges paper to Tengstrand structures with *P*=5-pass edge curves, skipping over the case *P*=4. It turns out that when *P* is even, things get more complicated. *P*=4 implies that each edge makes four passes past the central void; and this means that an individual edge starting from the top vertex makes a "down-up-down-up" move and then ends up again at the starting vertex (Fig.10d). All edges now start and end on the same vertices! This leads to an additional constraint. To preserve the symmetry of these sculptures, the number of edges terminating at the top vertex should be same as at the bottom vertex. Overall the symmetry is reduced by a factor of 2, since in each pyramid corner there are now only *E*/2 edge-curves present.

Figure 10 shows a working edge configuration for *E*=*P*=4. In Figure 10a, just the two top edges are shown, while in Figure 10b all four edges are depicted. Each individual edge-loop passes twice through one of the four branches, and once each through each of the two neighboring branches. This then leads to a handlebody composed of four ribbon-shaped faces that show a similar behavior: They each run from one of the inner junction areas through the two pyramids and then back to their starting point, thereby forming some kind of a 4-pass loop (Fig.10f), brushing tangentially against the blue junction patch (Fig.10g). But the edge configuration described above seals off the 4-sided junction area on two opposite sides. While all four ribbon-loops approach the two (blue) 4-way junction areas, only one pair of ribbons can join in one junction area. The other two pass the junction tangentially, separated from it by a sharp edge (Figs.10h,i).



(a) (b) (c) (d) (e)

Figure 10: *(a) General set-up of a "4-2-(?)"-sculpture, showing the two edges hooked to the top vertex. (b) All four edges shown resulting in a "4-2-3"-structure. (c) Bottom two edges rotated by 90 degrees, resulting in a "4-2-2"-structure. (d) A single edge-loop smoothed. (e) Smoothed "4-2-2"-model.*

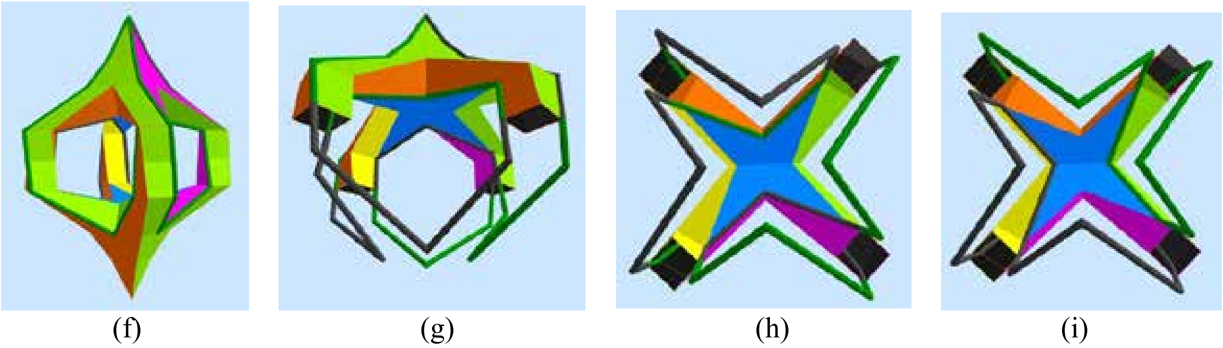

(f) (g) (h) (i)

Figure 10(ext.): *(f) Bi-pyramid frame with the two top 4-pass edges shown. (g) Inside view of top half, showing blue junction patch. (h) Edges placed to allow the yellow & lime ribbon loops to merge. (i) Edges rotated 90 degrees to allow the orange & magenta loops to merge.*

For the edge configuration shown in Figure 10b the same pair of ribbons will join in the top junction as well as in the lower one; this leaves two of the four ribbons isolated, and this results in a “4-2-3”-geometry. If the bottom two edges are rotated by 90 degrees (Fig.10c), one pair of ribbons merges in the top junction, and the other pair merges in the lower one; thus overall there are now only to *faces*, and the result is equivalent to a “4-2-2”-structure. Figure 10d depicts a single edge-loop smoothed with CC-subdivision; and Figure 10e shows this edge-loop in the context of the complete sculptural model.

Figure 11 shows pairs of topologically different sculptures. In Figure 11a I have painted the two merged *faces* in yellow and in green, respectively. When the bottom edge pair is rotated by 90 degrees, a “4-2-3”-structure results, since the red and the yellow ribbons are prevented from merging (Fig.11b).

Similarly, Figure 11c depicts a “6-2-2”-structure. If the three edge-curves hooked to the bottom vertex are rotated by 60 degrees, a “6-2-4”-structure results, because now three ribbons merge in both (white) 3-way junctions, while the other three ribbons remain isolated (Fig.11d).

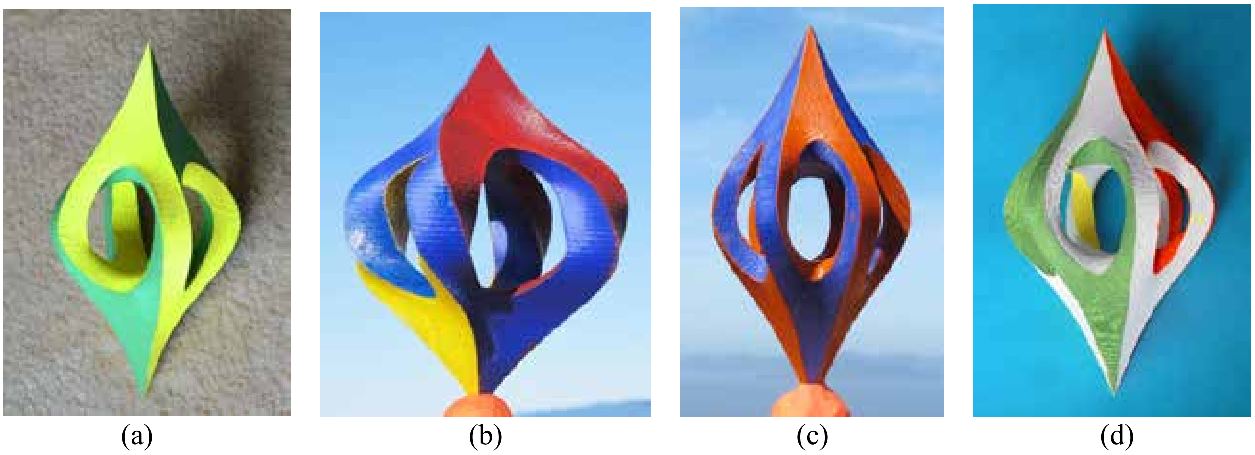

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Figure 11: *Painted models: (a)“4-2-2”shape; (b)“4-2-3”shape; (c)“6-2-2”shape; (d)“6-2-4”shape.*

Handle-Bodies of Genus-1 and “Arch” Modules

Somewhat surprisingly, the edge geometries and the behaviour of the “ribbon countries” discussed above can also be made to work on handlebodies of genus 1, with only two twisted prismatic branches between the two vertices. For the 3-pass edge-curve, this then results in a “2-2-1”-handlebody (Fig.12a).

When the two edges are joined at the tips of the two pyramids, we obtain a single curve that is topologically equivalent to a (2, 3)-torus-knot. For the more complicated *P*-pass edges with an odd *P*, the result is a (2, *P*)-torus-knot; for instance, the (2, 5)-torus-knot shown in Figure 12(b). For even values of *P*, we do not get the doubly covered (2, *P*)-torus-knot but two simpler (1, *P*/2)-torus-knots, which are out of phase by 180 degrees. Between them they accommodate two ribbon-like surface strips, shown in red and blue for *P*=4 (Fig.12c). We can contemplate what would happen if we used even simpler 2-pass edges: This would result in two interlinked (1,1)-Torus-knots. But this also implies that the two prismatic connecting branches are just 2-sided "prisms," i.e., flat ribbons (Fig.12d).

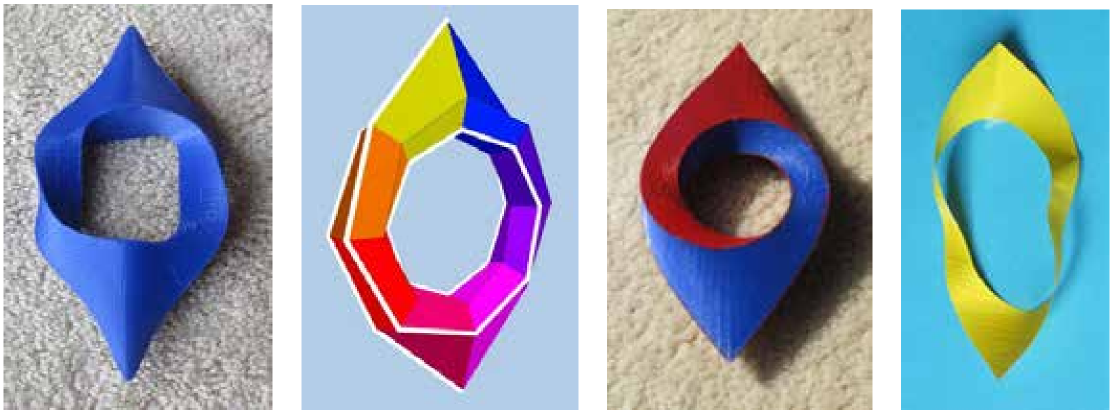

(a) (b) (c) (d)

Figure 12: *(a) "2-2-1"-structure leading to a (2, 3)-Torus-knot. (b) (2, 5)-Torus-knot. (c) 4-pass edges leading to two (1, 2)-Torus-knots. (d) Two linked (1, 1)-Torus-knots.*

The shapes presented in Figure 12 are of some modeling interest. I split them along the equatorial planes into two "arch"-like "half-modules" that form useful construction components (Fig.13a,d). An even number of these modules can be strung togther into an up-down zig-zag loop. Figure 13b displays a zig-zag loop with a square footprint built from four arch modules with legs with a triangular cross-section; this required a slight adjustment of the twisting of the two legs. It is more natural to fit six of these arches into a zig-zag ring with a hexagonal footprint (Fig.13c). On the other hand, four arches with square legs (Fig.13d) can readily be assembled into a ring with a square footprint (Fig.13e); it has been painted to show that this model has four individual ribbon-like faces.

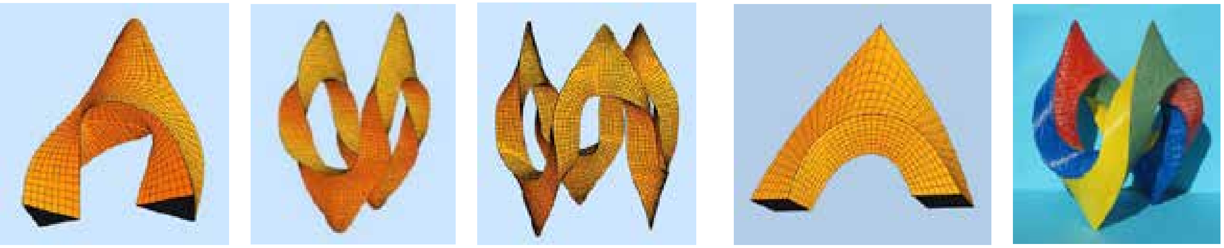

(a) (b) (c) (d) (e)

Figure 13: *(a) Half of Fig.12a. (b) Four such arches form a zig-zag structure with a square footprint. (c) 6-arch zig-zag structure with hexagonal footprint. (d) Half of Fig.12c. (e) 4-arch zig-zag ring.*

Combining "Legged-Pyramid" Half-Modules

The above constructions prompted me to also split the higher-genus structures into pairs of *B*-sided pyramidal "half"-modules with *B* legs, and then connect these modules into more complicated frame structures based on the Platonic polyhedra. Figute 14 shows the example of a half-module derived from the original Tengstrand sculpture with 3-pass edges on a handlebody with 3 branches (Fig.5b). I wanted to place four such modules at the corners of a tetrahedron and join their legs.

In order to obtain flush connections between the legs of neighboring “half”-modules, I introduced additional geometrical parameters into th three legs. I made the triangular prisms at the end of each leg adjustable in diameter, azimuth, and twist, and I added the freedom to adjust the tilt and the separation of these prisms to spread the three legs of the pyramid more broadly (Fig.14b). It was not surprising that four such “half-modules could be assembled into a tetrahedral frame (Fig.14c), and that, with the right edges marked as *sharp* (Fig.14b), the whole frame can be CC-subdivided [1], and the sharp edges would appear in the right places (Fig.14d).

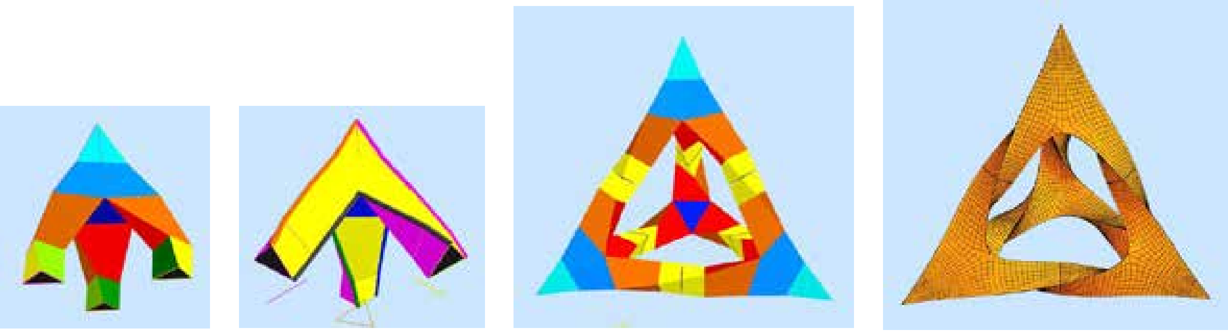

(a) (b) (c) (d)

Figure 14: *(a) Half of Figure 5b. (b) “Half”-module with “flexible” legs and specified sharp edges. (c) Tetrahedral assembly of four “half”-modules; (d) after CC-subdivision with sharp edges.*

A remaining question was: What kind of edge-pattern would emerge in the overall tetrahedral structure? It turns out that there are just six separate, smooth, “loopy” edge curves. Each one follows a contorted path that starts at one pyramid vertex, passes on the inside of two other corner pyramids, and then ends on the fourth vertex (Fig.15a). I would not have been able to design directly such a path based on a B-spline. It is interesting to note that the 3-pass edge in the bi-pyramid handlebodies turns into a “3-segment” sharp edge on the tetrahedral frame.

A similar question concerns the ribbon-like portions of the resulting single “face.” There are six separate ribbon countries. They start at one internal 3-way junction patch, pass through two pyramid faces, and then end at the junction inside the fourth pyramid. Figure 15b shows one such ribbon country painted on the tetrahedral frame. Six of those placed properly at the six edges of the tetrahedron completely cover the underlying handlebody (Fig.15c). However, all six ribbons are smoothly connected to one another through the four inner (black) junction areas, so that they form a single Tengstrand-style *face*.

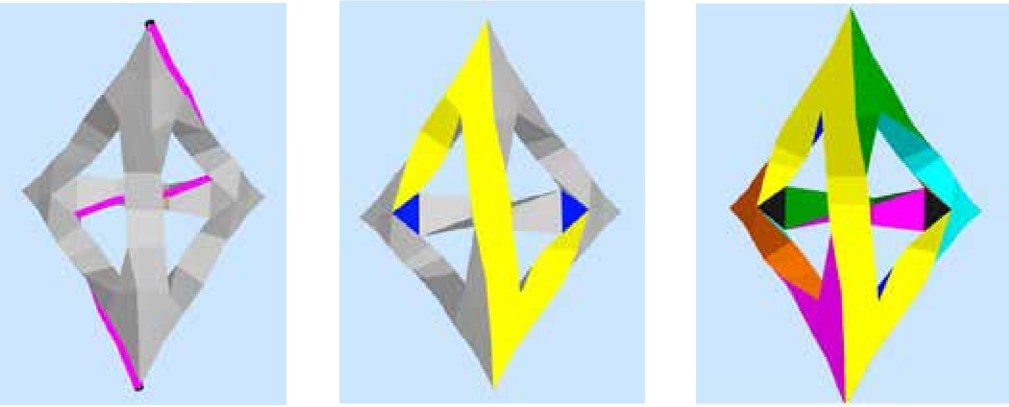

(a) (b) (c)

Figure 15: *Tetrahedra with: (a) one 3-segment edge. (b) one 3-segment ribbon. (c) six ribbons.*

In the same spirit, a somewhat differently deformed 3-legged corner pyramid can be used to construct a cuboidal frame (Fig.16a). Here we find twelve edges (Fig.16b) and twelve ribbon countries with a behavior similar to that in the tetrahedral frame. However, as a special case for the cuboidal frame, a single half-module using three colors, and its mirror image can be assembled in a way that produces a consistent coloring with only three colors for all twelve ribbon areas (Fig.16c). This is because among all five Platonic polyhedra, only in the cube can the two types of vertices (mirrored and non-mirrored) be placed in such a way that the two neighbors across any leg-joints are always of different types.

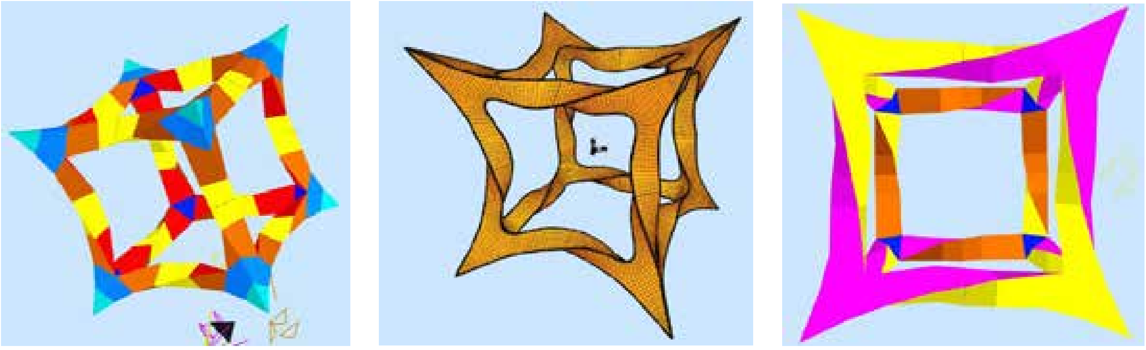

(a) (b) (c)

Figure 16: *(a) Cuboidal frame. (b) Twelve 3-segment edge-curves stretching from vertex to vertex. (c) Twelve surface ribbons painted with only three colors.*

For the construction of an octagonal frame (Fig.17d,e), we need six valence-4 pyramid half-modules with four legs. A polyhedral starting module (Fig.17a) can readily be obtained by splitting Figure 6c along the equatorial plane. Its legs are then spread to accommodate an octahedral frame (Fig.17b) and its edges are marked with appropriate sharp specifications (Fig.17c). The assembly of six half-modules (Fig.17d) is then subdivided and converted into an STL-file to fabricate a 3D-print (Fig.17e).

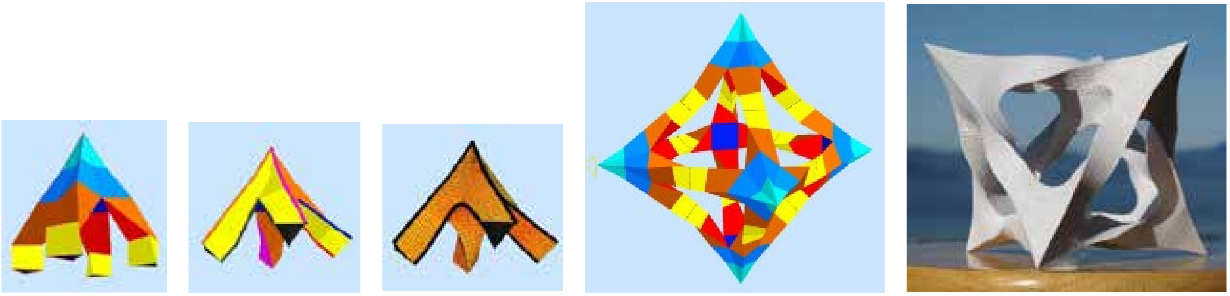
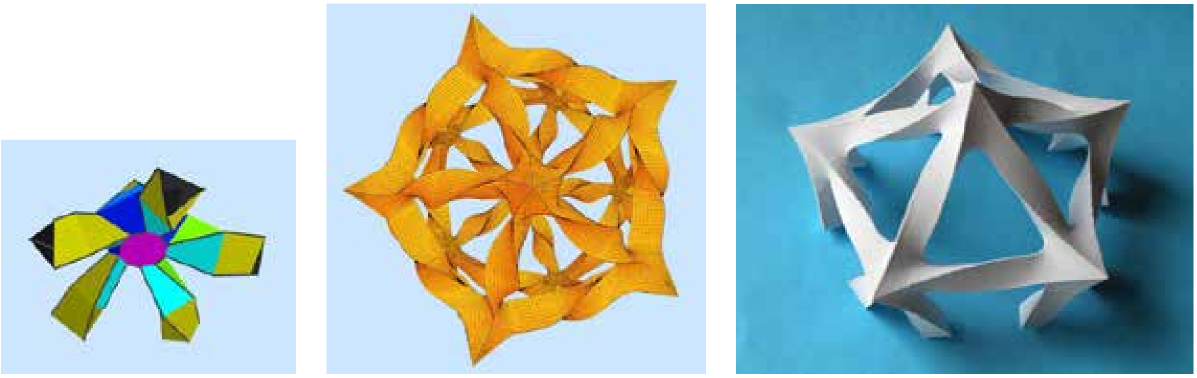

(a) (b) (c) (d) (e)

Figure 17: *(a,b,c) Valence-4 pyramid corner modules. (d) Six pyramid-modules assembled into an octahedral frame. (e) 3D-print of the octahedral assembly.*

Figure 18 shows the equivalent constructionof an icosahedral frame using twelve valence-5 pyramid corner modules. When trying to make a 3D-print of this rather dense frame structure (Fig.18b) on an inexpensive printer [8], one encounters the problem of having to remove a large amount of support material trapped in the interior of the frame. It is thus advantageous to to print this object in two parts (Fig.18c); this requires less support, and the support material is easier to remove. The two half-parts can then be glued together.

In the same way one can readily build a dodecahedral model by assembling twenty 3-legged corner modules.



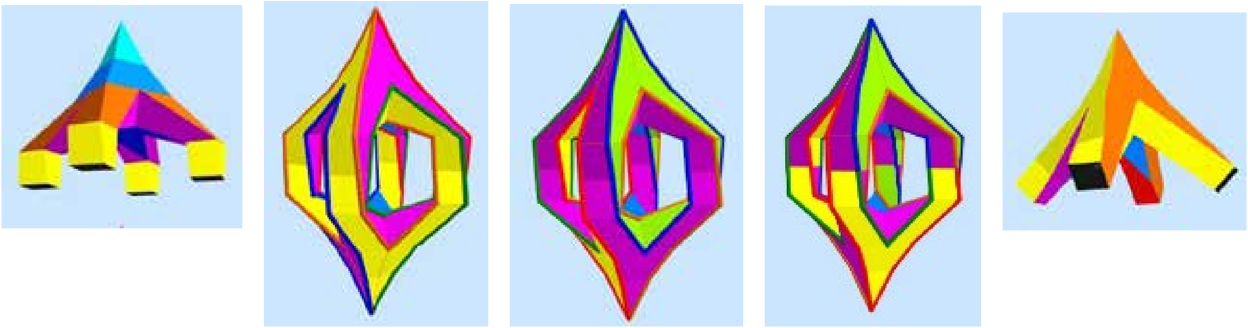
(a) (b) (c)

Figure 18: *(a) Valence-5 pyramid corner module. (b) Twelve pyramid modules assembled into an icosahedral frame. (c) 3D-print of half of an icosahedral frame.*

Platonic Frames with 4-Pass Edge-Curves

All five Platonic frames can readily be constructed with corner-modules based on 3-pass edges. A more interesting investigation is to look at half-modules derived from more complicated *P*-pass edge-curves; in particular the ones with an even value of *P*.

Let's take a close look at an octahedral frame based on a 4-pass edge-curve. I started by splitting Figure 11(b) into two half-modules (Fig.19a) and combining two of those into a bi-pyramid structure (Figs.19b,c,d). On this structure I study the behavior of the four 4-pass edges and the emerging ribbon-like surface elements. As shown in Figures 11(a) and 11(b), depending on which ribbons are allowed to merge in one of the two junction areas, the handle-body might be covered with either two (Fig.19b) or three (Fig.19c) separated *faces*. I now use the upper half of either one of these two structures as a 4-leg pyramid module to construct an octahedral frame composed of six such modules; I spread the four legs and adjust their tilt angle to 45 degrees (Fig.19e) to result in smooth leg-joints. Unfortunately, assembling six such modules does not automatically lead to contiguously colored ribbon segments; and even by rotating individual modules through steps of 90 degrees I could not produce a consistent, contiguous coloring. Similar to the problems discussed for the tetrahedral frame, the reason is that the legs of three modules forming a triangular cycle have an odd number of binary color changes and we always end up with one leg-joint with mismatched colors. The 4-pass edges do not solve this problem either.



(a) (b) (c) (d) (e)

Figure 19: *(a) Half of Fig.11b. (b) 2-color bi-pyramid. (c) 3-color bi-pyramid, (d) lower half turned 90°. (e) Half-module adjusted to serve as a corner of an octahedral frame.*

However, I was still able to extract the general behaviors of the edges and ribbons in an octahedral frame built from these pyramid modules: Similar to Figure 15(a), each edge starts at the outer tip of a pyramid module, but it now consecutively passes on the inside of three further modules, and then ends up at the tip of another pyramid module. This can be considered some kind of a “4-pass” move; and I will call this a “4-segment” edge. Figure 20a depicts five such 4-segment edges drawn on the polyhedral octa-frame. The red edge is rotated around the *z*-axis (perpendicular to the image plane) in steps of 90 degrees to generate the other three edges starting from the front (blue) vertex; and it is rotated 90 degrees around the *x*-axis to produce the dark blue instance of the 4-segment edge. In total, there are twelve such edges.

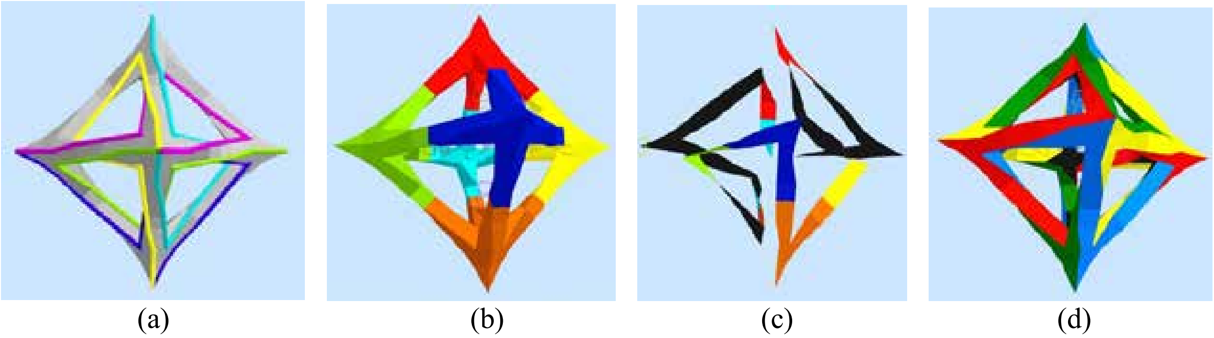

(a) (b) (c) (d)

Figure 20: *(a) Octa-frame with five 4-segment edges highlighted. (b) Color-coded pyramid-modules. (c) 12-segment ribbon-loop passing twice through every corner. (d) 4 loops form an octa-frame.*

The ribbons follow a more complicated pattern. Without the use of any mergers through any (black) junction patch, the ribbon areas form four 12-segment loops. In Figure 20b, I have colored the six pyramid modules differently to keep track of the travel of an individual ribbon loop (Fig.20c). The loop passes through every pyramid-module twice. It makes two consecutive inner passes that lead directly from one junction patch to another one. Then it passes through two outer pyramid faces, where it makes sharp hairpin turns before it tangentially touches another junction patch. This pattern repeats three times to complete the 12-segment loop. Four such ribbon loops assemble into a complete octa-frame (minus the six black junction patches) (Fig.20d).

Thus, before the role of the junction patches is considered, the total number of smoothly connected faces is already reduced to 4. In addition, at every junction patch one pair of ribbon-loops that touch the junction patch on opposite sides will merge through that junction patch, while the other two opposing loops remain isolated from the junction patch by two sharp edge segments.

As individual pyramid-modules get rotated in 90-degree steps, the pattern of the ribbon faces remains the same, but the connectivity between the four 12-segment loops changes. When I chose the original rotations to put the six pyramid-modules at the vertices of an octahedron, I treated them as if they had full 4-fold symmetry, and I paid no attention to their locally reduced symmetries due to the junction patches that are bordered on two of their four sides by sharp edges. Further investigation is required to find out what combinations of these six independent rotation parameters will lead to an overall octahedral frame with the highest degree of symmetry. However, every color combination appears at least once in one of the six junction areas, so this should make it possible to merge all four loops into a single, smoothly connected *face*. This would then result in a “12-6-1”-Tengstrand structure.

Summary

In Tengstrand’s original “3-2-1”-sculpture, <u>three</u> identical *edges* wind around the three *branches* connecting the <u>two</u> *vertices* in a 3-sided by-pyramidal frame structure. These three edges delineate only <u>one</u> single, complex, smoothly connected “*face*,” embedded in the surface of a genus-2 handlebody. My exploration started with an investigation of what happens when the number of edges, *E*, is changed from *3* to another small integer number, *E*. In the first derivative designs, I maintained *E*-fold rotational symmetry around an axis that connects the two vertices, and I also maintained the *E* connecting branches as 3-sided, twisted prismatic rods (Fig.6). This generalization was rather straightforward, and it always resulted in a handlebody of genus (*E*–1) with a single *face* covering the whole surface.

I subsequently studied what happens when I use more complicated edge-curves that make more than three passes past the central void in the bi-pyramid. To accommodate *P*-pass edges (and *ribbons*) I also increased the number of polygonal sides in the prismatic branches to *P*. Now *P* edges and *P* ribbon segments pass through every branch (Fig.9). When *P* is an even number, the situation becomes more complicated. The resulting edges then start and end at the same vertex. This constrains the number of edges, *E*, to be even; and this reduces the resulting symmetry of the resulting sculpture by a factor of two. Moreover, depending on how I arranged the edge-loops hooked to the two corner pyramids with respect to one another, the number of resulting *faces* could be changed, but the minimum number of faces was always two (Fig.11).

I also used half of these bi-pyramidal frames as pliable *half-modules* to generate more complex symmetrical frame structures. In all the resulting handlebodies, I kept all the edges and all the corner structures exactly the same. When *E* was set to *2*, I could connect several of these *arch*-shaped half-modules into closed-loop chains of genus-1. For instance, six such modules can be connected into a zig-zag structure with a hexagonal footprint (Fig.13c). The pyramid corners with more than two legs were also used to construct highly symmetrical frames based on the Platonic polyhedra (Figs.14-18).

The structures analyzed and discussed in this paper have two key parameters, *E* and *P*, specifying the number, *E*, of *sharp*, *P*-pass edges. The number of physical, prismatic branches, *B*, is always the same as the number of sharp edges, and the number of polygonal sides in these branches is equal to *P*, accommodating the *P* "segments" of the zig-zag-shaped edge curves. Similarly, the resulting 2D surfaces of these physical objects are composed of *E* zig-zag-shaped *P*-segment *ribbons*, which are connected to some degree in the *V* junction areas below the *pyramid* corners of the overall frame structure. This is true for the simple bi-pyramid structures as well as for the Platonic frames.

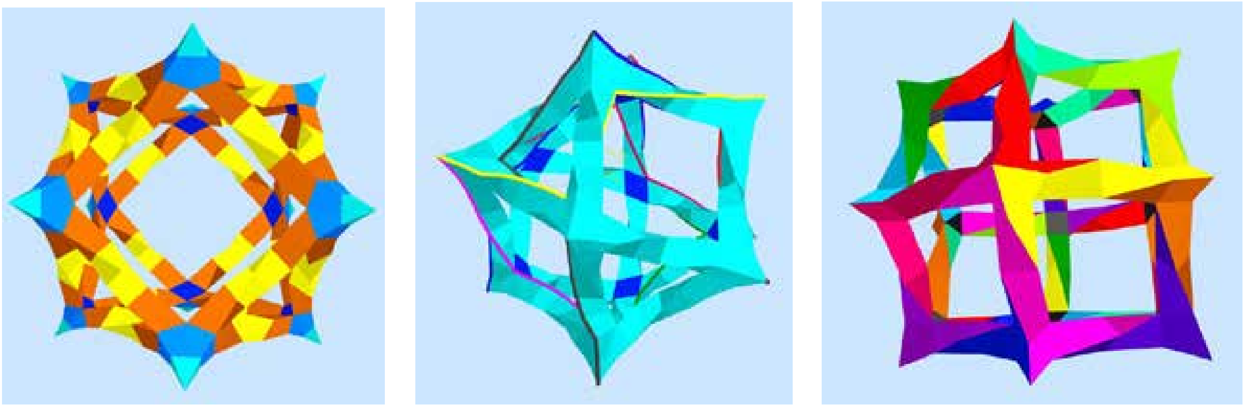
Conclusions

Tengstrand's "*edges-vertices-faces*"-notation is primarily a topological characterization of bi-pyramid handlebodies covered with a mesh of sharp edges. In choosing specific geometries to introduce my new derivative models, I designed shapes that could depict the 3D results easily through 2D images, and which also stayed true to the "character" of the original Tengstrand sculpture.

To decide whether a new handlebody still belonged into the *Tengstrand Family*, my primary criterion was whether all the edge-curves had the same shape. This criterion readily includes all the bi-pyramid designs and the Platonic frame structures. The issue became more intricate when I started to explore some Archimedean frames. Most of these frames clearly fall outside the boundaries of the *Tengstrand Family*, because not all the edges have the same pair of neighboring faces.

But the cuboctahedron (Fig.21a) and the icosidodecahedron are exceptions; here all edges and vertices are the same. However, the corner modules no longer have 4-fold rotational symmetry; they assume a more rectangular pyramid shape with only 2-fold rotational symmetry. For 3-pass edges this is good enough to give all edge-curves the same shape (Fig.21b) and keep this frame in the *Tengstrand Family*.

A more daring move is to look at the duals of these two Archimedean polyhedra. They both comprise two different types of vertices with valences 3 & 4, and 3 & 5, respectively; and this might already put them outside the bounds of the *Tengstrand Family*. On the other hand, on the rhombic dodecahedron (Fig.21c), every 3-segment edge-curve starts on one type of vertex (valence 3) and ends on the second type (valence 4), thus the edges can still be all the same. The same is true for the 3-segment ribbon structures. Thus, a rhombic-dodecahedral frame with twisted prismatic edges can still be considered to lie in the *Tengstrand Family*.



(a) (b) (c)

Figure 21: *(a) 3-pass cuboctahedron; (b) overlaid with ten colored copies of a 3-pass edge-curve. Rhombic dodecahedron painted in 12 different colors to show the 24 3-segment ribbon countries.*

In summary, joining twisted prismatic beams in symmetrical configurations can lead to intriguing geometrical shapes. I am still surprised that the original, inspiring sculpture by Tord Tengstrand has led to so many different shapes and presented me with many puzzles and design challenges.

Acknowledgment

I am grateful to Tord Tengstrand for presenting his "3-2-1"-sculpture at Bridges 2020, and for involving me in a lively email discussion about its geometry and its topology. This allowed me to do the fascinating exploration presented in this paper.

References

[1] E. Catmull, J. Clark. "Recursively generated B-spline surfaces on arbitrary topological meshes." – https://people.eecs.berkeley.edu/~sequin/CS284/PAPERS/CatmullClark_SDSurf.pdf

[2] G. Dieppedalle. "Interactive CAD Software for the Design of 2-manifold Free-form Surfaces." Tech-Report (EECS-2018-48). – https://www2.eecs.berkeley.edu/Pubs/TechRpts/2018/EECS-2018-48.pdf

[3] R. Fan, C. H. Séquin, R. Ng. "Joining Interactive Graphics and Procedural Modeling for Precise Free-Form Designs." Tech-Report (EECS-2021-125) May 14, 2021. – https://www2.eecs.berkeley.edu/Pubs/TechRpts/2021/EECS-2021-125.html

[4] JIPCAD Language Reference. – https://tinyurl.com/9ds62ee2

[5] C. H. Séquin. "Handle-Bodies Inspired by Tengstrand's "3-2-1"-Sculpture." Accepted for Bridges 2024.

[6] J. Smith. "SLIDE design environment." (2003). – https://people.eecs.berkeley.edu/~ug/slide/

[7] Tord Tengstrand "3-2-1-Sculpture." Bridges 2020 Art Exhibit – https://gallery.bridgesmathart.org/exhibitions/2020-bridges-conference/tordtengstrandteliacom

[8] Ultimaker. "Reliable 3D printers." – https://ultimaker.com/3d-printers

A Unifying and Productive N-dimensional Fractal Algorithm

Manuel Diaz Regueiro

Retired Math Teacher, Spain; mdregueiro2@gmail.com

Abstract

The Menger-Diaz 3D fractal algorithm, published in Hyperseen [2], now presents its potential as an N-dimensional fractal algorithm. This algorithm serves as a unifying system, generating well-known fractals in both 2D and 3D. By modifying the existence matrix, it produces diverse fractal forms.

The study extends to n-dimensional generalization, with a particular focus on 2D and tesseracts in the fourth dimension. All of this explained with programming examples, analyzing the parameters, providing code examples to generate fractals in Rhino Python, highlighting the flexibility and simplicity of the algorithm.

.

Introduction

The algorithm, which we introduced for the first time at SCHULPT 2023 and later in Hyperseeing 2023, serves as a unifying system. It harmonizes various fractals into a single generative framework. By manipulating the existence matrix, this algorithm can generate diverse fractals,including the triangle, carpet, Sierpinski tetrahedron, Menger sponge, Vicsek fractal, and their variants.

Now, let’s explore its applications in two dimensions: It adapts seamlessly to different polygons. Furthermore, our study has expanded to n-dimensional generalization, with a particular focus on tesseracts in the fourth dimension. This advancement could unlock new possibilities within fractal geometry. However, achieving n-dimensional visualization requires novel representation systems capable of depicting dimensions beyond three.

The Menger Sponge

The Menger Sponge represents a three-dimensional extension of the one-dimensional Cantor set and the two-dimensional Sierpinski carpet. It was originally described by Karl Menger in 1926 [1] during his exploration of topological dimensions.

The construction process for a Menger sponge unfolds as follows:

1. Begin with a cube.
2. Divide each face of the cube into nine smaller squares, akin to a Rubik’s Cube. This subdivision results in 27 miniature cubes.
3. Remove the central smaller cube from each face, as well as the central smaller cube within the larger cube. This leaves us with 20 smaller cubes, constituting a level-1 Menger sponge (resembling a void cube).
4. Repeat steps two and three for each of the remaining smaller cubes, continuing this iterative process indefinitely.

The Menger Sponge showcases remarkable self-similarity and complexity, making it a captivating object of study in mathematics and fractal geometry.

Recursion

Recursion is a process that invokes itself either directly or indirectly. The corresponding function is referred to as a recursive function. In this iterative approach, a rule or operation is repeatedly applied to a given set of elements. However, to prevent infinite recursion, a stopping criterion is essential. Typically, only the level of recursion changes during each iteration.

The complexity of recursion varies based on the rules we apply. As we introduce increasingly intricate rules, the process becomes more intricate as well.

These complex rules, while flexible and easily modifiable, must be stored in the program. When rules alter the number of segments, squares, or cubes in each iteration, we need to track which elements exist at each step of the algorithm. In some cases, it suffices to store the rules for generating new elements in a matrix (referred to as the “existence matrix”). By retaining existing elements in this matrix, we can proceed to the next iteration. We can even consider this process as a form of “Menger iteration,” where the generated elements require a specific memory position.

Menger’s Algorithm

Menger’s algorithm stands as one of the earliest elegant examples that adheres to specific rules. Imagine taking a cube and dividing it, much like a Rubik’s Cube, resulting in 27 smaller equal cubes. Now, eliminate the 6 cubes located at the center of each face and the one at the center of the original cube.

The Menger cube, also known as the Menger sponge, played a pivotal role in extending and popularizing the concept of fractals due to its beautiful and straightforward description. However, its practical application as an algorithm can be somewhat intricate. This fascinating structure has spread worldwide—I’ve encountered it in places like the Science Museum in Santa Cruz de Tenerife and the Verbum in Vigo. Menger’s achievement remains universal and mesmerizing. While a single sentence can describe recursion, applying it effectively to a 3D program is no small feat. In certain versions, the new cubes are determined using a base-3 numbering system for the numbers 1 to 27.

Existence matrix

The concept of an existence matrix is crucial when dealing with recursive algorithms like Menger’s. Essentially, we need a set of rules to determine which cubes persist in each recursion and which ones do not. Consider a given level of recursion, denoted as “l”. At this level, we have a total of 27^l cube positions, and our task is to determine whether each position will ultimately be occupied or remain empty.

To achieve this, we establish rules for the permanence or exclusion of cubes generated during each recursion. If our goal is to replicate the Menger fractal, there are straightforward ways to memorize these rules. However, if we aim to generalize Menger’s algorithm by adjusting the number of segments into which each original cube edge is divided (represented by the variable *nsegm*), we need a more flexible approach.

Enter the existence matrix—a three-dimensional representation that encodes Boolean values (True or False, or 1 or 0) for each possible position. This matrix organizes the locations systematically, making it easy to reference the existence status of a cube at a specific place or its exclusion.

In summary, our task is to determine which positions are excluded or remain occupied during each iteration. Armed with this existence matrix, we can explore a multitude of new algorithms and shapes. The possibilities are vast, especially considering that the number of 3-dimensional matrices we can create is 2^nsegm^3.

This existence matrix applies not only to the classic Menger algorithm (where each cube is divided into sub cubes) but also to the Menger-Diaz algorithm. In the latter, each cube “multiplies” into many other cubes in space, following the same existence matrix rules.

While both methods are nearly equivalent, the Menger-Diaz approach requires rescaling at each iteration (or at the end). Implementing the second algorithm becomes more straightforward using tools like Rhino 3D, where we can apply copy instructions to objects in various locations determined by the existence matrix.

In essence, Menger represents a top-down design, while Menger-Diaz leverages existing object-copying instructions for a more flexible, bottom-up approach. By adjusting parameters like the distance factor (*d*), which ensures that polyhedra or polygons align at vertices, edges, or faces, we can create novel forms. Keep in mind that experimenting with these concepts—whether through Rhino or other 3D programs—offers deeper insights into their behavior.

Menger-Diaz Algorithm

The Menger-Diaz algorithm transforms Menger’s design into a parametric model, offering multiple options—a strength of Rhino design and Rhino with Grasshopper. To achieve this, we define the new parameters *nsegm* (number of segments) and *d* (separation).

Menger-Diaz is a more productive algorithm because it expands the application of the Menger algorithm by varying these parameters. The separation distance between objects depends on the specific figures we’re working with, often requiring experimental adjustments. For regular polygons, the radius parameter defines the size of the polygon. For instance, an octagon’s size is determined by the radius of the circle in which it is inscribed. Depending on orientation, we may also need to calculate the apothem of the octagon. As a general assumption for squares, we take radius = 50 and d = 35.

Units: We use the units available in the Rhino program (e.g., mm, cm, meters). Since we create pattern figures that are perfectly scalable while preserving the pattern, the visual result doesn't depend on the chosen units.

In summary, Menger-Diaz involves parameterization of the following:

1. Initial Atom: This may differ from a cube.
2. Number of Segments: Each edge of the cube is divided into segments.
3. Distance in each Iteration: Determines how the figure evolves.
4. Iteration Rules: These can lead to numerically large results.
5. Criteria for Beauty or Usefulness: Defining what forms are aesthetically pleasing or useful.
6. Handling Dimensions: We can modify Menger-Diaz for dimensions less than three and address challenges when dimensions exceed 3. The algorithm encourages thinking in n dimensions.

Adaptation of the algorithm to 2D

When transitioning from a 3D algorithm to a 2D representation, what changes do we need to make? Let's explore the necessary adjustments. We'll consider the following parameters:

1. Dimension of Euclidean Space (*dim*): This parameter defines the space in which the fractal is represented.
2. Number of Segments (*nsegm*): Each side of the square is divided into segments.
3. Recursion Level (*level*): Determines the depth of the fractal.
4. Separation Distance (*d*): Typically, *d* equals size of a segment multiplied by 2. d=Cube edge/nsegm*2.
5. *Radius* and *d* : These parameters play a role in shaping the fractal.
6. Program Simplicity: 2D programs tend to be simpler than their 3D counterparts.

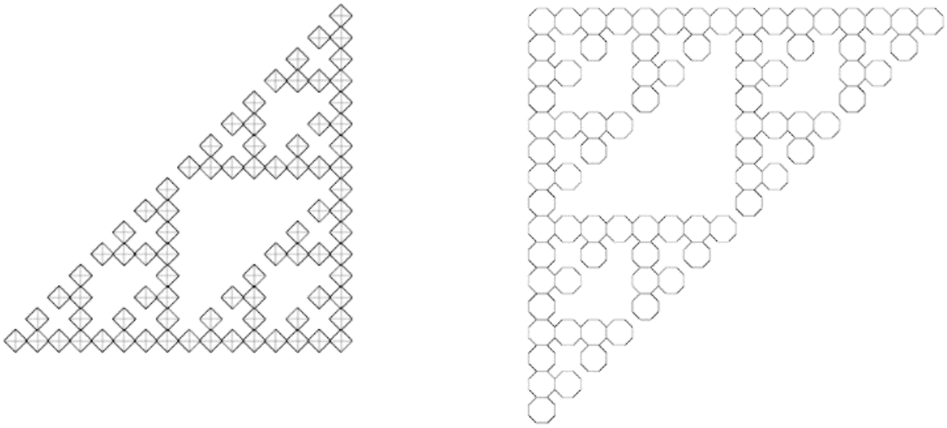
Advantages of 2D:

1. Ease of Review and Understanding: The reduced complexity in 2D makes these algorithms more accessible for review and comprehension.
2. High Number of Illustrations: To verify results, we often create numerous illustrations.
3. Challenges in 4D: In 4D, projecting results back to 3D becomes challenging, limiting our exploration to a single example.

I'll proceed to create the programs in Rhino Python, starting with a Sierpinski triangle.

```
import rhinoscriptsyntax as rs
nsegm=2
a=[[1,0],[1,1]]
#We draw a regular polygon in Rhino with 4 sides, squares rotated
# the z coordinates in the plane z=0, are 0
d=25
for level in range(1,4):
    objs=rs.AllObjects()
     for i in range(nsegm):
         for j in range(nsegm):
             if a[i][j]==1:
                 rs.CopyObject(objs,(i*d*nsegm**level,j*d*nsegm**level,0))
     rs.DeleteObjects(objs);
```

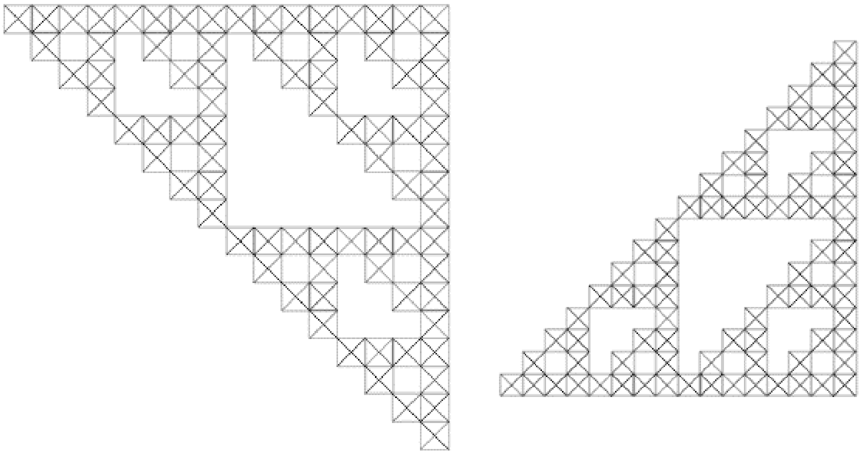
Figures 1, 2 are the result of changing a zero in the existence matrix. What will the figure be like if a=[[1,1],[0,1]] with octagons? In this case, *d* is related to the apothem of the octagon.



(a) a=[[1,1],[1,0]], d=25 (b) a=[[1,1],[0,1]], d=46.19

Figure 1: *Sierpinski fractal* (a) *With squares rotated (b) With octagons*

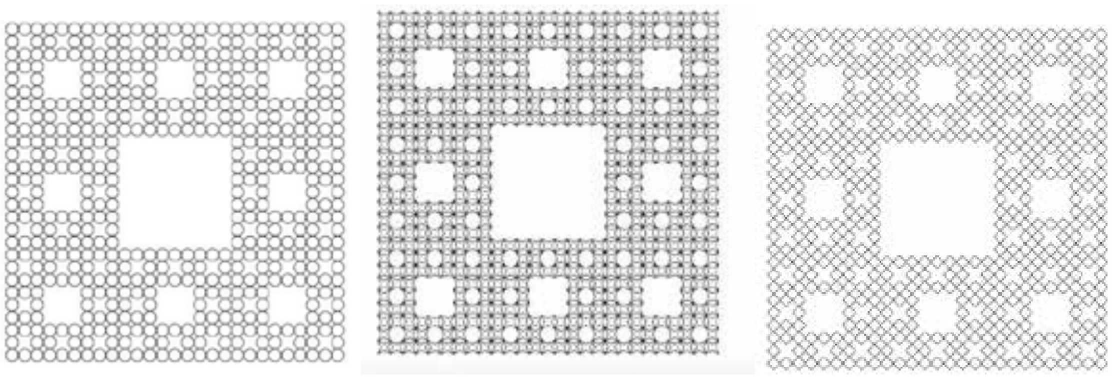
Variants of Sierpinski triangles with squares. Now radius=50, d=35



(a) a=[[0,1],[1,1]] (b) a=[[1,0],[1,1]]

Figure 2: *Sierpinski fractal with squares.*

2D Menger sponge or Sierpinski's carpet a=[[1,1,1],[1,0,1],[1,1,1]], nsegm=3



(a) (b) (c)

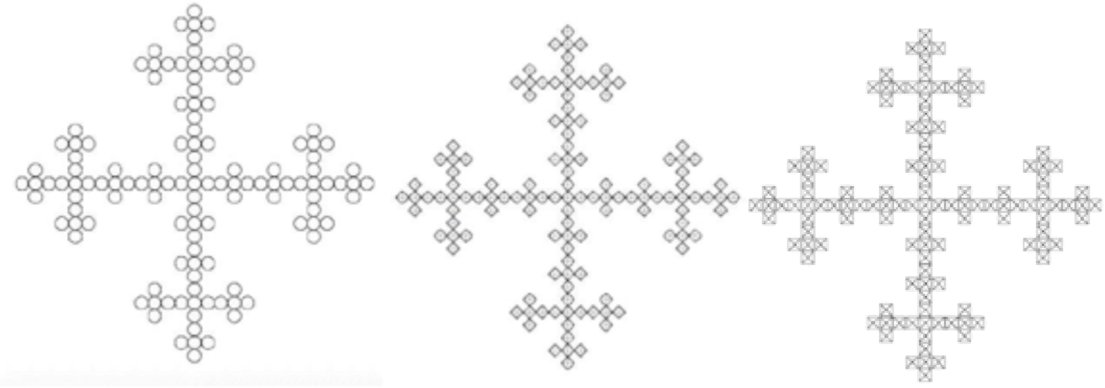
Figure 3: *Menger fractal* (a) *with octagons, d=33. (b)With four pointed stars (c) With square rotated d=32*

```
import rhinoscriptsyntax as rs
# octagon, 4 star, square, square rotated 90 n=3
d=32
for level in range(1,4):
     objs=rs.AllObjects()
     for i in range(nsegm):
          for j in range(nsegm):
             if a[i][j]==1:
                rs.CopyObject(objs,(i*d*nsegm**level,j*d*nsegm**level,0))
     rs.DeleteObjects(objs);
```

If we wanted to do with circles then we would use rs.AddCircle((0,0,0),radius) and, of course, d=radius.

Vicsek's fractal with octagons and with squares

a=[[0,1,0],[1,1,1],[0,1,0]] His 3d equivalent is Mosely [9]



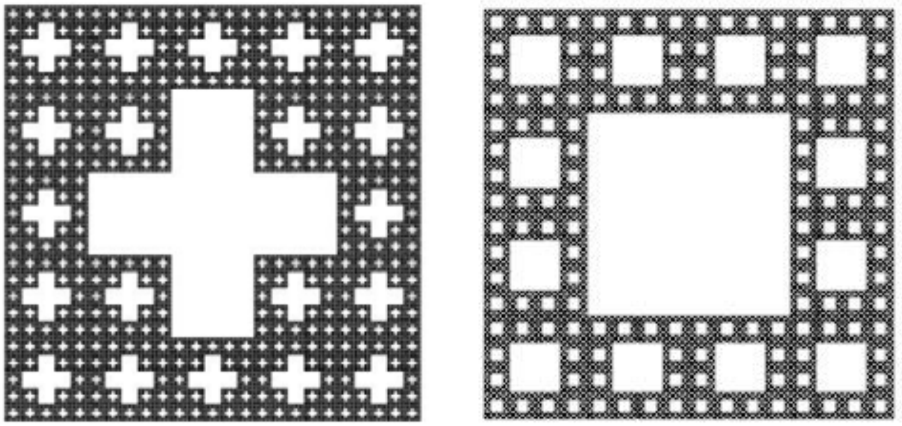
(a) (b) (c)

Figure 4: *Vicsek fractal (a) with octagons (b) with rotated squares (c) and squares*

Now we do nsegm>3
Let's look at something more complicated, the cross:

Cross in 2D

a=[[1,1,1,1,1], [1,1,0,1,1], [1,0,0,0,1], [1,1,0,1,1], [1,1,1,1,1]]
nsegm=5, d=14, level=3



(a) (b)

Figure 5: *Sierpinski's carpet Cross (a) and Great Hole4 (b)*

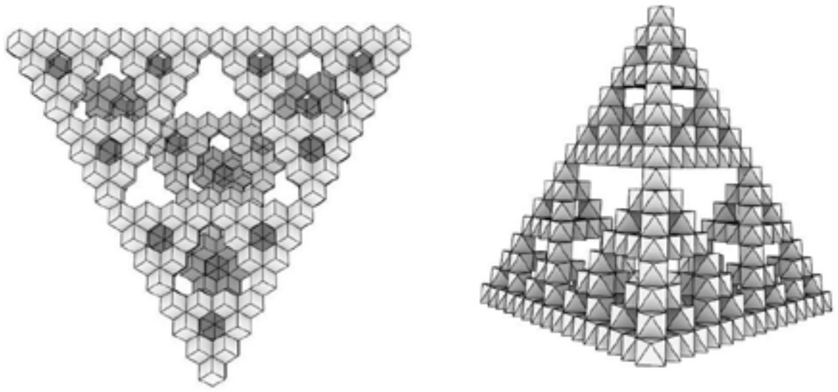
Great hole 4(b), nsegm=4, a=[[1,1,1,1],[1,0,0,1],[1,0,0,1],[1,1,1,1]]

The algorithm 3D

In PythonScript of Rhino for a Sierpinski tetrahedron, but is the same for any list of nested vectors changing the variable *a* and the size of cube. For each level you must repeat the algorithm changing the variable *level.*

```
import rhinoscriptsyntax as rs

objs = rs.AllObjects()

level=1 # the level

d=30#distance. its depends on polyhedron and their size

a= [[[1,0], [0,1]], [[0,1], [1,0]]]

for i in range(2):

    for j in range(2):

        for k in range(2):

            if a[i][j][k]==1:

                rs.CopyObject(objs,(i*d*2**level, j*d*2**level, 5k*d*2**level))

rs.DeleteObjects(objs);
```

You can see numerous examples of Menger-Diaz 3D in [2] and [3], including all 3x3 symmetric face cubes.



(a) (b)

Figure 6: *Sierpinski tetrahedron with (a) Rhombic dodecahedron (b) Cuboctahedron*

Differences between Menger and our Algorithm

The traditional algorithm divides a cube into smaller cubes respecting the shape and eliminating certain subdivisions. Our algorithm does not divide the cube but places it at predetermined distances that depend on n^l , where *l* is the level and *n* is the size of the units of the cube. Enlarges rather than divides the figure (although we are supposed to eventually scale or shrink to the size of the first figure at level 0)

To understand the distinction between our proposed algorithm and Menger's, we need to consider the digital world and how easily elements can be copied within it.

In our digital day-to-day, we continuously copy and reposition elements. Menger, in his pre-digital or analog world, envisioned a top-to-bottom approach—dividing a figure by extracting its predefined parts recursively. For this purpose, he used an easily divisible shape: the cube. In our proposal, we take a different path. We start from the bottom (with the first brick) and construct the figure using Menger recursion.

Our approach involves building rather than separating. At each iteration, the resulting figure becomes the "brick" for the next step. This flexibility allows us to create an algorithm with only a few lines of code, applicable in various situations. Here's the basic process:

1. **Lay the First Brick:**
    - Start with an initial shape (the "brick or atom").
    - Place this brick in a loop.
2. **Position Copies:**
    - Calculate the distance.
    - Position copies of the initial shape at predefined points within the loop.
3. **Repeat the Loop:**
    - Convert the resulting figure back to the initial shape from step 2.
    - Repeat the process.

In Menger's Algorithm the number 3, the cube, and 7 empty spaces are fundamental. Some daring ones that we mention in the bibliography keep the number 3, the cubes, and change the position of the empty spaces. We do not have all the possible bibliography, which, in reality, is more extensive, so we cite the best known, even the recommended ones, but that we were not previously aware of.

In our algorithm, we focus on the cube as the primary shape (for better understanding). We also create images of shapes with *nsegm*=3, not to emphasize them as our main contribution, but to facilitate comparison with familiar concepts. Specifically, we are dealing with cubes composed of nsegm^3 smaller cubes, where *nsegm* ranges from 2 to 3 (similar to Menger's approach). Refer to the article [2] for further details.

This exploration leads to an infinity of possible forms. For instance, when nsegm=6, there are 2^6^3 potential fractals—more than the number of atoms in the Sun. Representing all these fractals on paper would be impossible (and not all of them would be aesthetically pleasing).

And what if we don't work within a cube? Consider other shapes like rectangular prisms, stepped pyramids, or any three-dimensional network of points. The challenge lies in determining placement positions for each copied element in every iteration. There is plenty of exciting work ahead for those who want to explore it further!

Into the Fourth Dimension and beyond

There is a Bridges 2023 article on the Menger cube in the fourth dimension and this is the origin of this article. In that article they present mathematical ideas about the sections and their visual result, but not an algorithm to build it.

The construction of n-dimensional Menger cubes is very simple with our algorithm:

-First of all, the existence matrices are larger (we already saw how the existence matrices in 2D are smaller than in 3D).

-Secondly, the algorithm only changes with the number of *for* lines (one for each dimension), so if in 2D we have 2 *for* lines, in 3D we have 3 *for* lines, in 4 dimensions we will have 4 *for* lines, etc.

```
for x in range(nsegm):
    for y in range(nsegm):
        for z in range(nsegm):
            for w in range(nsegm):
```

-Thirdly, the coordinates in 4 dimensions are 4 and that must be reflected in the instruction `rs.CopyObject(objs,(x*d*nsegm**level,y*d*nsegm**level,z*d*nsegm**level, w*d*nsegm**level))`, instruction that does not exist in Rhino, obviously, and it is necessary to do the calculations of the chosen 3D projection of the 4d fractal.

The problem is that Rhino does not display 4 dimensions, but it does save in Obj files, which do save points in 4 dimensions. So progress on this topic should go in the direction of having 4D figure viewers in 3D for certain figure orientations (which exist in the form of Python programs or Unity programs), and in the coding of objects of 4,5, 6,..., n dimensions as files that store their n-dimensional characteristics. And the existence of viewing programs for these n-dimensional shapes. However, we can create 4D visualizations of tesseracts if we think of it in a Dalí Cross-style unfolding of the hypercube in 3D representation.

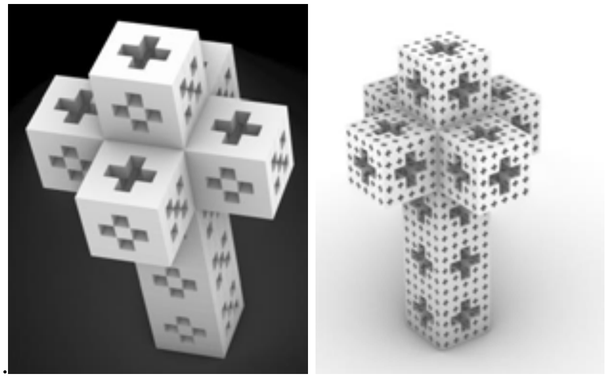


Figure 7: *Menger-Diaz Cross tesseracts in Dalí Cross style net.*

Conclusions and new advances

Numerous algorithms exist for generating Sierpinski triangles, Menger cubes, and their variants- possibly hundreds. However, none is as unifying as the one presented here. Our algorithm, simple and concise, can be applied to various geometric objects (polygons or polyhedra) and adapted to different positions (such as horizontal and vertical squares) with minimal effort. The key lies in adjusting numerical parameters, such as d.

Wolfram's more general algorithm [4] (found on page 188) addresses square conversion, but it shares some conceptual similarities with the existence matrix definition. Despite this, it lacks implementation for the *nsegm* parameter, and its mystical quality can obscure understanding.

Unlike Wolfram's approach, our algorithm seamlessly integrates into 2D or 3D design programs like Rhino without requiring a complete reinvention of the software. However, the mystique surrounding the fourth dimension poses challenges for n-dimensional geometric visualization.

In our algorithm, each dimension corresponds to a straightforward "for" loop in the program. Yet, the problem of displaying geometric objects in dimensions greater than 3 remains unsolved. Similarly, saving n-dimensional geometric objects in standardized formats (yet to be defined) represents another avenue for future exploration.

As we venture into higher dimensions, we embrace both the beauty and complexity that await us. The path forward involves not only mathematical insights but also practical solutions for visualizing and preserving these intricate forms.

References

[1] K. Menger, *Dimensionstheorie*. 1928.

[2] M. Diaz and L. Diaz . An Algorithm for Extending Menger-Type Fractal Structures. Hyperseeing 2023

[3] M. Diaz and L. Diaz . Variations on Menger-Díaz Fractals

https://easychair.org/publications/preprint/QFNq69

[4] Wolfram, Stephen. *A New Kind of Science*. Wolfram Media, 2002.

[5] C. A. Reiter, “Sierpinski fractals and GCDs,” *Comput Graph*, vol. 18, no. 6, pp. 885–891, Nov.1994, doi: 10.1016/0097-8493(94)90015-9.

[6] Benoît B. Mandelbrot, *Fractals: Form, Chance, and Dimensions*. San Francisco, 1977.

[7] Y. D. Sergeyev, “Evaluating the exact infinitesimal values of area of Sierpinski’s carpet and volume of Menger’s sponge,” *Chaos Solitons Fractals*, vol. 42, no. 5, pp. 3042–3046, Dec.2009, doi: 10.1016/j.chaos.2009.04.013.

[8] Eric Baird, “The Jerusalem cube,” *Magazine Tangente*, p. 45, 2013.

[9] L. Wade, “Folding Fractal Art from 49,000 Business Cards,” *Wired*, 2017. Accessed: Dec. 28, 2022. [Online]. Available: https://www.wired.com/2012/09/folded-fractal-art-cards/

[10] George W. Hart. Procedural Generation of Sculptural Forms. https://archive.bridgesmathart.org/2008/bridges2008-209.html

[11] https://demonstrations.wolfram.com/SubstitutionSystemsIn3D/

[12] https://blogs.mathworks.com/cleve/2022/01/15/the-menger-sponge-complement-and-associate/

[13] SCHULPT 2023 Show and Tell July

Tracing Lines of Force: Structural Adaptations and Material Variations in Helical Column Designs Across Multiple Scales

Mark L. Donohue

Architecture Department
California College of the Arts
San Francisco, CA 94107

Abstract

This paper presents an investigation into the formal and structural relationship among three architectural objects inspired by the helical form of Solomonic columns. Despite their similar form, each object employs distinct approaches to resolve structural stability issues, influenced by the materials used and the scale of the structure. The analysis focuses on the interplay of compressive and tensile forces within each object, tracing the lines of force as they move through space. Through analysis of three examples (one in marble, another in wood struts and string cables, and a third which is a combination of wood, steel, and concrete elements) the paper explores the diverse approaches taken to resolve structural stability while embracing the aesthetic allure of spiraling geometry. By tracing the lines of force inherent in these designs, the paper uncovers the intricate interplay between materiality, construction techniques, and spatial expression. This study contributes to a deeper understanding of the design principles underlying helical architectural forms and their practical implications across different scales.

Keywords: Solomonic Column, Helical Geometry, Tensile and Compressive Forces

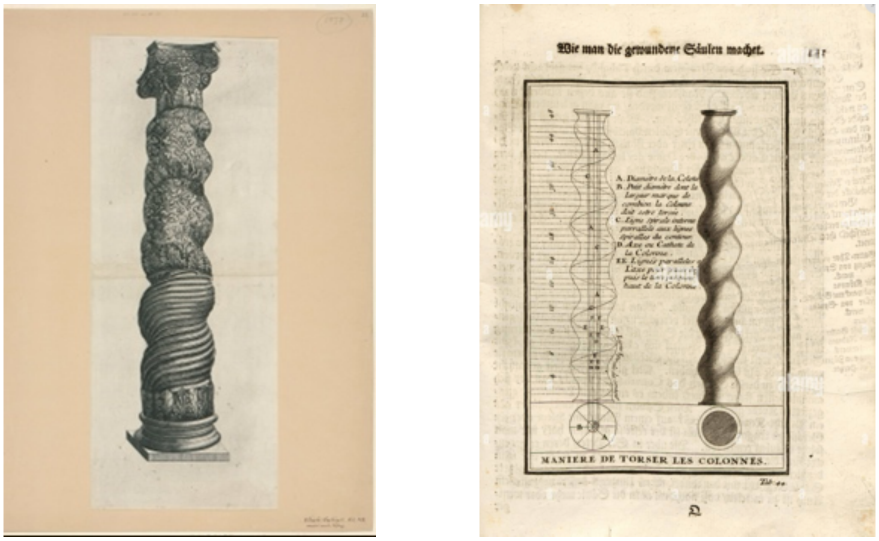
1. Introduction

The Solomonic column [1], characterized by its twisting helical form, has long fascinated architects and engineers due to its aesthetic appeal and structural complexity. From ancient monuments to contemporary landmarks, architects have continually sought to explore and reinterpret this iconic motif in innovative ways. A close examination of Solomonic columns can show how this motif continues to be important through time as a way to connect to the past while moving into the future. Perhaps one of the best-known examples is the column for Bernini's Baldacchino in St. Peter's Basilica in Vatican City (Fig. 1a). Completed in 1633, these composite multistory bronze columns allude to the spiral columns used by Constantine, the first Christian Roman emperor, in a basilica dedicated to St. Peter in 333AD. The construction of the baldachin required that the Constantine columns be moved. In their new location the Constantine columns became additions to the pilasters (see upper right image Fig. 1a) that support the central dome of St. Peter's and a visual referent for the baldachin column origins. Bernini's choice of the Solomonic column was two-fold in that it connected his work to that of the founding of the church while the S-shaped profile gave the column a dynamism which energized the work clearly alluding to the Baroque times of which he was a part.

In addition to its historic importance, the Solomonic column's geometric form, its changing perception as one moves around it, the way it is constructed in segments, and its potential as an occupiable space are all important to the work presented here. The segmented construction can be seen in a painting by Raphael (Fig. 1b) and in a drawing (Fig. 2a) from the same time period. Raphael's "The Healing of the Lame Man" from 1515 which uses the column as a setting and compositional device to organize the depiction of St. Peter's first public miracle. The columns frame the activity and place the action in front of the Temple in Jerusalem's Porta Speciosa (Beautiful Gate). Columns have not only been used as structural elements to hold up parts of buildings, on certain occasions they have been used as occupiable spaces. An example of this can be seen in tower design attributed to Vignola (Fig. 2b). The drawing published in 1725 clearly depicts in elevation a tower with the defining S-shaped profile of the Solomonic column while ignoring what makes the plan different than a straight column. While a cylinder is the geometric shape that underlies all columns, the Solomonic column differs in plan as the circle extrudes vertically to become a cylinder. In a Solomonic column, the circle rotates about an axis which is not at the center of the circle as it moves vertically. This produces the helical or corkscrew shape associated with the Solomonic column. Changes to the location of the axis and number of rotations modify the resulting helical form produce a wide array of possible shapes. This variation and malleability of the columns is important to my work presented here.

(a) (b)

Figure 1: *Solomonic columns found in the history of architecture and painting: (a) Bernini, baldachin in St. Peter's Basilica, (b) Raphael, "The Healing of the Lame Man".*



(a) (b)

Figure 2: *(a) Unknown Artist, Print, The Column from the Temple of Solomon (1570–73). (b) Giacomo Barozzi da Vignola, "Come rendere le colonne a spirale, costruzione di una spirale di forma colonnare", Tab. 44, p. 121, Sturm, Leonhard Cristiano (1725).*

Taking the rich expressive potential of the Solomonic column as its starting point, this paper delves into three contemporary interpretations of the column, each addressing structural stability through different material choices and construction techniques. The materials used to make each of the three pieces varies according to the structural load requirements to yield a self-supporting shape. The first object ‘Carrara Serra’ (Fig. 3a) reflects compressive forces and is made of stacked marble slabs. The second ‘Air Lines’ (Fig. 3b) uses two distinct and separate systems of construction to resolve tensile and compressive forces at an intermediate scale through the use of wood struts, wood plates, and string cables. The third and most complex piece ‘Twist Light Tower’ (Fig. 3c) uses various combinations of wood, steel, and concrete elements to resolve compressive and tensile forces at a large scale. By analyzing Carrara Serra's marble slabs, Air Lines' string lines, and the Twist Light Tower's hybrid structure, this research aims to elucidate the relationship between form, materials, and structural performance in helical column design.

(a) (b) (c)

Figure 3: *Comparative analysis showing the structural adaptations and material variations across the three designs by Mark Donohue: (a) Carrara Serra, (b) Air Lines, (c) Twist Light Tower.*

2. Carrara Serra: Exploring the Boundaries of Material and Form

Carrara Serra serves as an exploration into the possibilities of water jet cutting technology in sculpting 2” thick marble slabs into intricate helical forms. I did the work presented here while I was participating in Autodesk’s Artist in Residence program at Pier 9 in San Francisco in 2018. I was inspired by the work of Richard Serra’s Torqued Ellipse [2] and Constantin Brancusi’s Endless Column [3], the torqued elliptical form, which can be understood as a continuous ruled surface, pushed the 5 axis limits of an OMAX water jet cutter. Four different types of pieces were produced (Fig. 4a-d): a transformation of a circle to an ellipse, and three ellipses with varying degrees of rotation (0°, 45°, 90°). The pieces can be stacked in an endless number of configurations to produce a column of varying shape (Fig. 5a-d). Structurally speaking Carrara Serra is no different than the marble columns that were used to build the Parthenon. The age-old technique of adding stone upon stone using compression to resist the force of gravity. By manipulating the geometry of the torqued ellipse, Carrara Serra demonstrates the potential for marrying ancient materials with modern manufacturing technology.

The difficulty in making the piece was learning how to get the OMAX water jet cutter to cut at an angle other than vertical. OMAX water jet cutters are preset to cut vertically at 90° degrees to the cutting surface. The machines does this by mapping X and Y coordinates of the top surface with those of the bottom connecting the dots so’s to speak to make the right sized cut depth (Z coordinate). What I found is that this could be altered such that the pattern of X and Y coordinates on the top surface could be different than the pattern of X and Y coordinates on the bottom. This allowed the nozzle to cut at an angle. After some trial and error I was able to get the machine to make a cut at a single angle. With additional time I was then able to input directions which would allow for a continuously adjusting angled cut. This in turn produced the ruled surface cut I was looking for to make helical shaped pieces.

Beyond its technical achievements, Carrara Serra embodies a deeper inquiry into the nature of form and space. By abstracting the helical column into a series of modular components, this project invites viewers to engage with the architectural form in new and unexpected ways. The endless configurations made possible by these interchangeable elements evoke a sense of dynamism and fluidity.

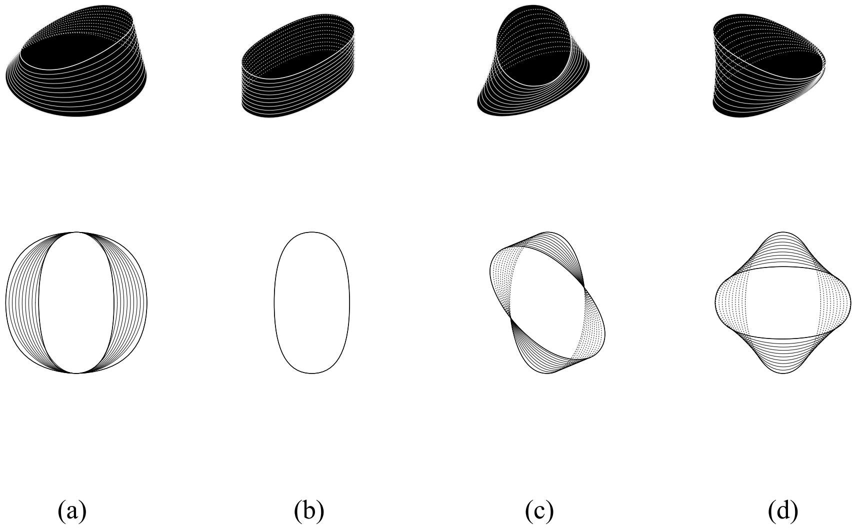


Figure 4: *Carrara Serra's marble slab variation: (a) circle to ellipse, 0°, 45°, 90° (L-R)*

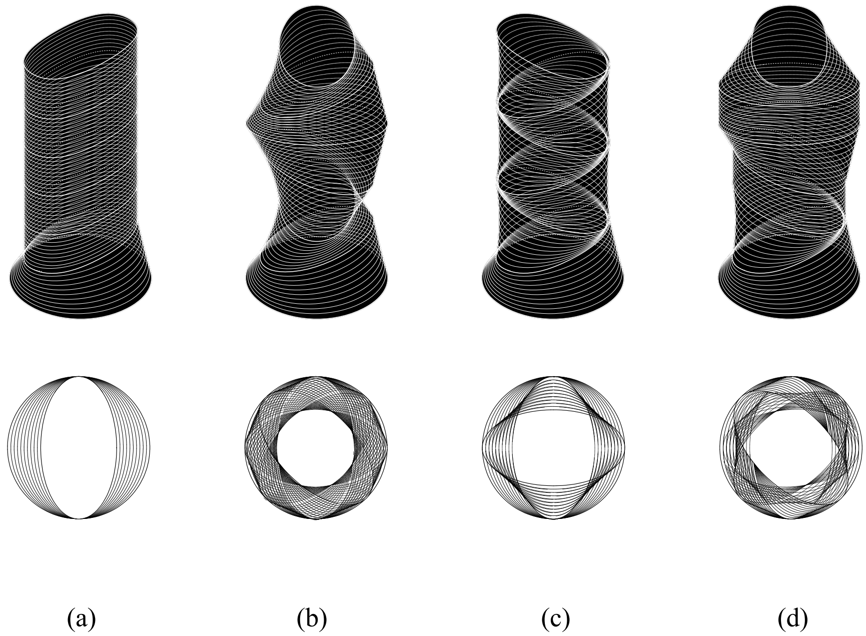
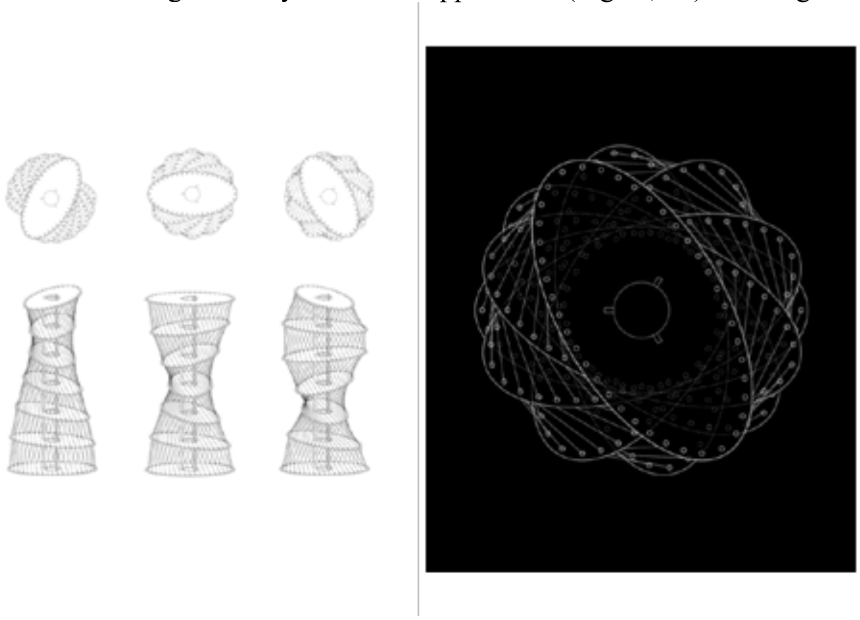


Figure 5: *Carrara Serra's stack variations: (a) circle to ellipse + 5x0°, (b) circle to ellipse + 5x45°, (c) circle to ellipse + 5x90°, (d) mixed*

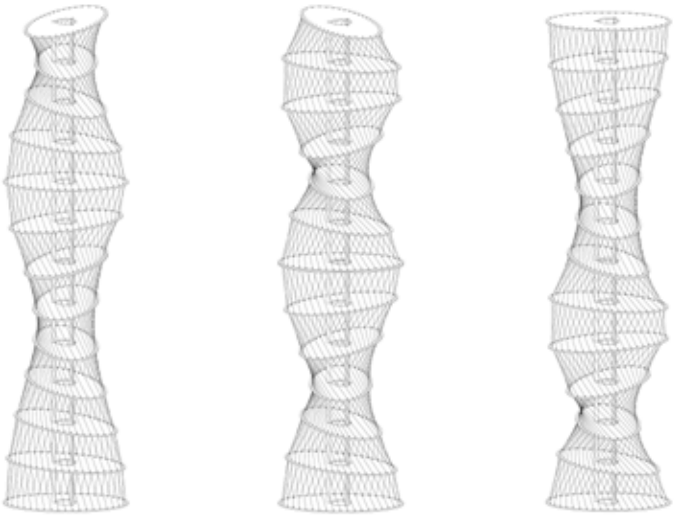
3. Air Lines: Bridging the Divide Between Materiality and Immateriality

This prototype for a light fixture picks up where Carrara Serra left off translating the straight lines cut by water jet through marble into string lines suspended in air. I did this work as a speculative project for a local light manufacturing company in 2021. As with Carrara Serra, the piece relies on the rotation of an ellipse as the basis for its helical form. Helical form is based on concentric congruent ellipses that are rotated about the center at equal intervals such that the ellipses are externally tangent to the circle in which they are inscribed. Rotating a vertical array of seven elliptical layers at either 20°, 30°, or 40° increments (Fig. 6a) yields three parts with string line patterns (Fig. 6b) unique to each piece. Variety in profile and reading of the piece is introduced through interchangeable stacking nature of the parts giving it any number of possible configurations allowing it to vary its overall appearance (Fig 7a, 7b) and height from 18” to 36”.



(a) (b)

Figure 6: *Air Lines prototype with 18” stackable parts: (a) axonometric view of 20°, 30°, 40° rotation (L-R), (b) plan view of 40° rotation showing string lines*



(a) (b) (c)

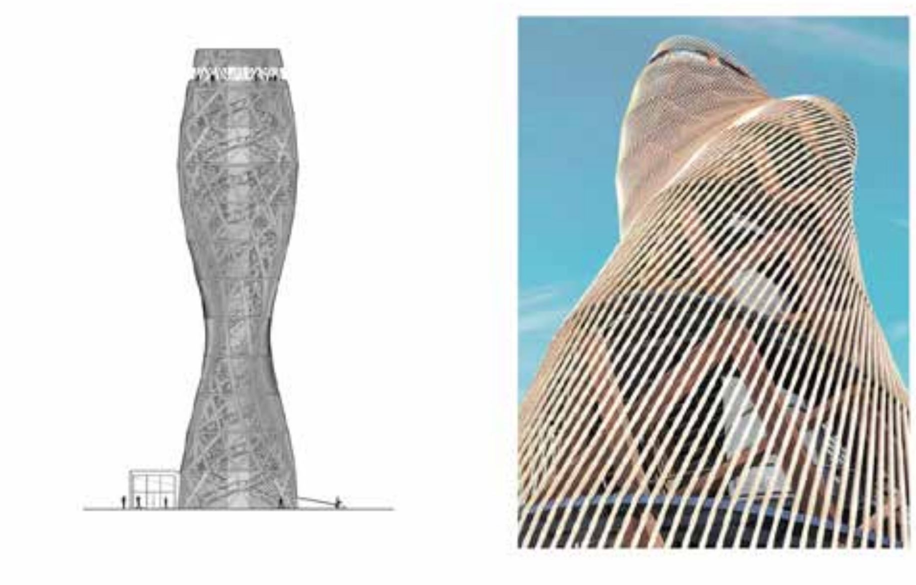
Figure 7: *Air Lines prototype with two stackable parts (36” tall): (a) 20° + 30°, (b) 30° + 40°, (c) 40° + 20°*

At the heart of Air Lines lies a delicate balance between tension and compression, lightness and solidity. The helical form, once synonymous with the weight and mass of stone, is reimagined here as a delicate web of tension cables and structural struts, suspended in a delicate dance of equilibrium. As light filters through the translucent layers of the helical structure, it casts intricate patterns of shadow and illumination, blurring the boundaries between physical form and ephemeral experience.

As one moves around the piece, the outer profile comes alive constantly shifting in shape as string lines advance, overlap, and retreat. The gentle spiraling shape of the piece bears a formal similarity to a Solomonic column where light and air have been substituted for heavy masonry. Reading the piece as a column is further heightened by seeing the string lines as an interpretation of fluting and the change in profile width as entasis. A stable structure is achieved through the interaction of compressive struts at the center with string tension cables around the perimeter. The torsional forces are balanced within the price by the number and configuration of the compressive struts in the center of the piece. The compressive struts are also placed at a slight angle to begin with which gradually gets straightened out by adding the tension string around the perimeter. The wood plates ensure that the torsional forces are distributed evenly between each of the compressive struts and the surrounding string cables to help eliminate accumulation of loads in any one member.

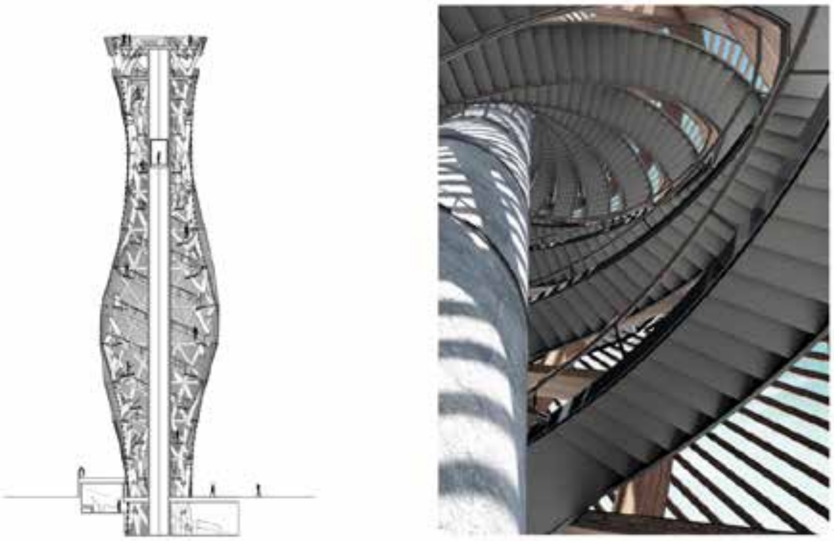
4. Twist Light Tower: A Monument to Innovation and Sustainability

Twist Light Tower is my entry for the 2022 Urban Confluence international design competition. The program brief asked for the design of an iconic tower for Silicon Valley to be located in a park along the Guadalupe River in San Jose California. My proposal is an architectural response to the technological innovations of Silicon Valley through structural daring and cutting-edge wood construction. My proposal follows a lineage of structural daring and innovation that started with the San Jose Electric Light Tower [4.] and its European cousins the Eiffel Tower in Paris in addition to Vladimir Shukov's [5.] [6.] various hyperboloid structures in Russia that tested the limits of steel construction to achieve maximum height with limited materials. If the 20th century built a legacy of observation towers in steel and concrete that achieved record heights, it has only been in recent years that towers have been constructed using wood as a primary structural component. The proposed hybrid structure would have a core of concrete and an exterior diagrid of glu-lam timbers with steel hoop rings and tensile cables all sheathed in a sustainable wood screen (Fig. 8a, 8b, 9a, 9b).



(a) (b)

Figure 8: *Twist Light Tower showing diagrid clad in wood lattice: (a) elevation drawing, (b) exterior view from below*



(a) (b)

Figure 9: *Twist Light Tower showing interior of tower: (a) section showing concrete elevator core and spiral stair, (b) interior view looking up through stair and concrete core*

The word twist has within its definition material and structural implications when acting on physical objects. In its most elemental form, twist is the uniting of two or more strands by winding or coiling them together. This happens quite naturally when a torsional stress is applied to an object by simply turning the ends of the object in opposite directions. Shape alteration results from compressive and tensile forces at work within the object such as yarn causing the threads to mingle by interlacing thereby increasing its strength. The degree to which an object twists is directly dependent upon the angle through which a thing is twisted.

A 20˚ rotation in twelve increments brings the Twist Light Tower to life. It appears different from all angles due to this simple maneuver (Fig 10). The tower's elliptical plan was chosen for its ability to enhance the reading of the displacement of volume appearing wider from some angles and narrower from others.

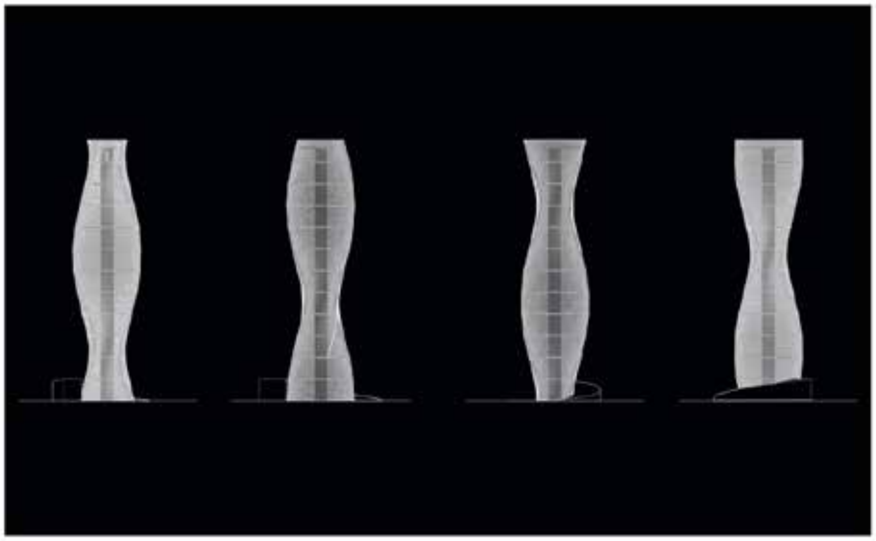
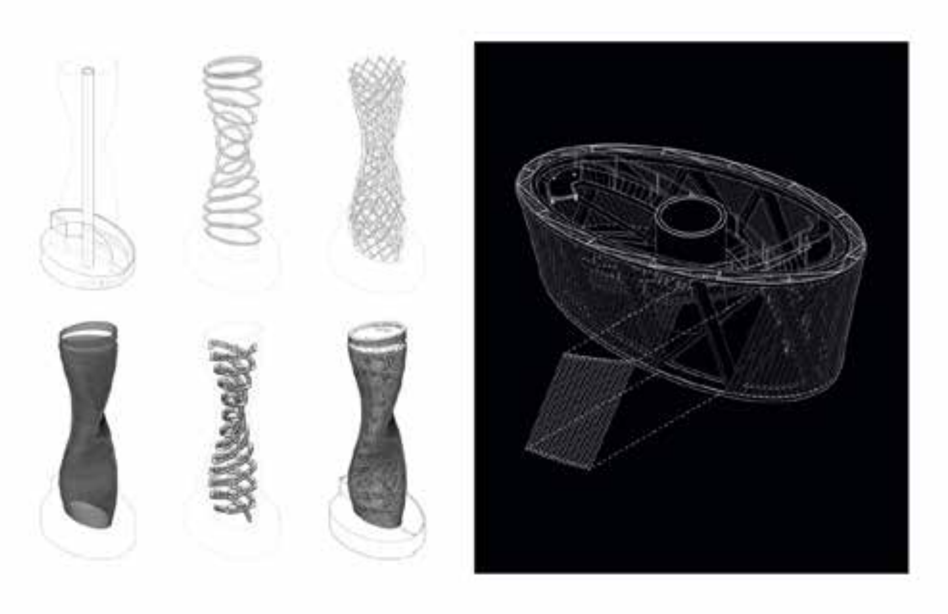


Figure 10: *Twist Light Tower as seen in elevation from various angles around the tower.*

Rotating the ellipse in an upward spiral motion creates a visual effect. When one moves around the tower it appears to be dynamically shifting its weight spiraling from bottom to top. The architectural form of twist light tower was modelled mathematically using a Rhino (a 3D modeling program) and scripting using Grasshopper (a parametric modeling plug in designed for Rhino). A series of digital models were generated to test various degrees of rotation for both expressiveness in form and economy of means. The 20˚ rotation seemed optimal. This had the added advantage of rotating the tower 240˚ enabling it to align itself with the park and adjacent streets below while at the same time aligning diagonally with the panoramic view of downtown San Jose from the observation deck above.

Like a piece of rope or yarn that has been made by twisting, Twist Light Tower is constructed of layers of interlaced parts (Fig. 11a). The inner layer is a concrete elevator core which has a circular stair wrapped around it. The outer layer is a hybrid structure made of glu-lam wood, tube steel and metal cables. The glu-lam members act as a truss which circles back to join itself forming an elliptical shape. The top and bottom cords of the truss made of tube steel form rings rotated 20˚ from each other. The precise angle of rotation produces a skewed ruled surface that solves several problems. It aids in stability while enabling the straight members to pass one another before meeting at the structural nodes on the steel ring. To construct the tower, the concrete elevator core is poured and the elliptical trusses are placed around it in successive levels. Each elliptical truss is stacked on-top of the one before it. Steel cables are installed after the trusses have been stacked. They form diagonal connections between the structural nodes of each successive layer of steel rings. The steel cables act in tension to laterally brace the entire tower while gravitational forces are carried through the glu-lam trusses that act primarily in compression. A wood screen made of dimensional lumber unifies the exterior. It is installed as the final layer to the tower in panels attached to each level of the truss (Fig. 11b).



(a) (b)

Figure 9: *Construction of the tower: (a) (top L-R) concrete core, steel rings, diagrid, (bottom L-R) wood lattice, spiral stair, composite, (b) elliptical truss made of multiple layers clad in wood lattice.*

5. Conclusion

Through the comparative analysis of Carrara Serra, Air Lines, and the Twist Light Tower, this research paper has explored the diverse approaches to helical column design across multiple scales. From the traditional craftsmanship of marble slabs to the innovative use of string lines and mass timber, each interpretation offers valuable insights into the interplay between form, materials, and structural performance. As architects and engineers continue to push the boundaries of design, the Solomonic column captivating both the eye and the mind with its dynamic form and structural complexity remains a timeless motif, inviting reinterpretation and innovation in the built environment.

6. References

[1.] R. Taylor, “Solomonic columns in England: the origins and influence of the porch of St Mary the Virgin, Oxford”, The Georgian Group Journal, Vol. XXIV, 2000, pp1-12
[2.] R. Serra, “Torqued Ellipse I”, 1996, Dia Art Foundation
[3.] C. Brancusi, “La Colonne sans fin (Endless-column)”, version 1, 1918, MOMA
[4.] L. S. Larson, “San Jose’s Monument to Progress: The Electric Light Tower”, San Jose Historical Museum Association, 1989
[5.] E. C. English, “Vladimir Shukhov and the Invention of Hyperboloid Structure”, Structures Congress 2005: Metropolis and Beyond
[6.] E. Nozhova, U. Hassler, “Networks of Construction: Vladimir Shukhov”, Hirmer Publishers, 2019

Robotic Luminous Seashells

Ali Farajmandi

Email: Ali.ferji@gmail.com
Website: https://ferjii.myportfolio.com/
Date Created: December , 2023
Dimensions: *30 × 30 × 30 cm (approx.).*
Materials Used: Light

Description

My Robotic Luminous Seashells project explores the connection between natural forms and technology, inspired by the growth process of mollusks. Mollusks create their intricate spiral shells layer by layer, similar to 3D printing. Using computational modeling in Grasshopper, I translated the logarithmic spirals and geometric principles of shell growth—expand, rotate, and twist—into robotic motion paths.
To visualize these designs, I built an LED tool attached to an ABB IRB 120 robot arm, programming it to trace the shell's form. Through long-exposure photography, I captured luminous patterns in dark settings, stacking multiple 5-second exposures into single images. These radiant visualizations represent not only the shells' natural geometry but also the robotic precision in replicating them.
This project bridges art and science, creating visually striking compositions while exploring the conceptual possibilities of robotic fabrication and nature-inspired design.

Statement

The Robotic Luminous Seashells project reimagines the growth process of spiral seashells through robotic accuracy and artistic expression. Inspired by mollusks' natural shell formation, I used computational modeling to design geometric paths. Attaching an LED tool to a robot arm, I captured the dynamic spirals with long-exposure photography, creating luminous visualizations that celebrate the mathematical elegance of nature.
This work exemplifies the fusion of technology and organic design, emphasizing both aesthetic beauty and the innovative potential of robotic fabrication. It highlights how art and science can converge to transform and elevate natural forms in creative and meaningful ways.

Designer(s) Biography:

Ali Farajmandi is an architectural designer and CCA graduate with expertise in computational design and kinetic structures. With a foundation in Landscape Design and a Master's in Advanced Architectural Design, Ali combines creative problem-solving with technical precision to push the boundaries of architectural design. With experience at Autodesk Technology Center, he has honed skills in digital fabrication and transformable structures. Passionate about collaboration, he thrives in interdisciplinary teams, exploring the intersection of architecture and landscape to deliver innovative solutions. Ali's work focuses on reimagining spaces through adaptable, dynamic, and forward-thinking design approaches.

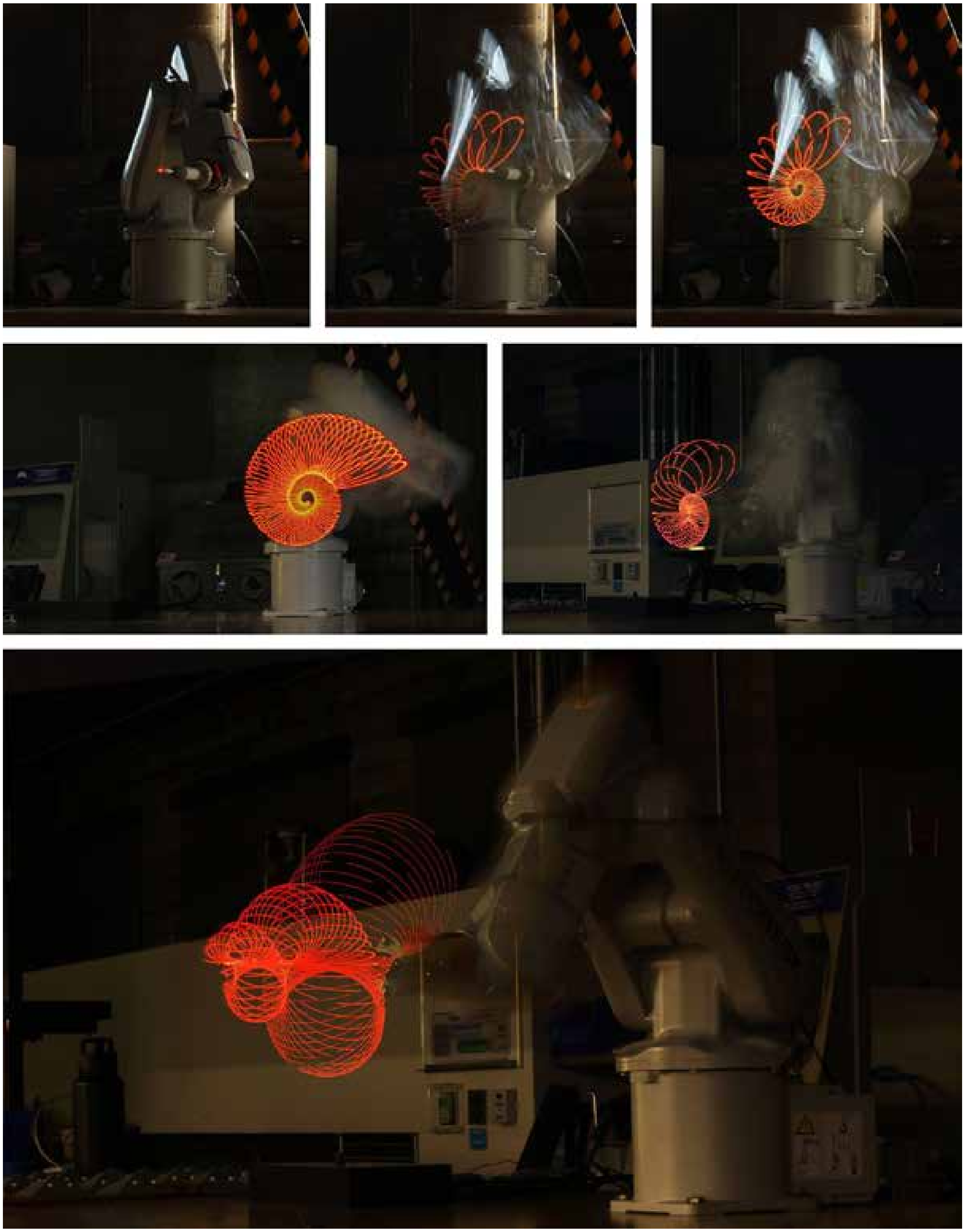


Figure. (Top two rows) Visualizations of spiral seashells constructed using the growth principles of Expand and Rotate. (Bottom) Visualizations created with a combination of Expand, Rotate, and Twist.

The complex interplay of mathematics and art through borosilicate glass

Anduriel Widmark

Email: anduriel@andurielstudios.com
Website: https://www.andurielstudios.com/

Date Created: 2024
Dimensions: 18x10x10 in.
Materials Used: Borosilicate Glass

Statement

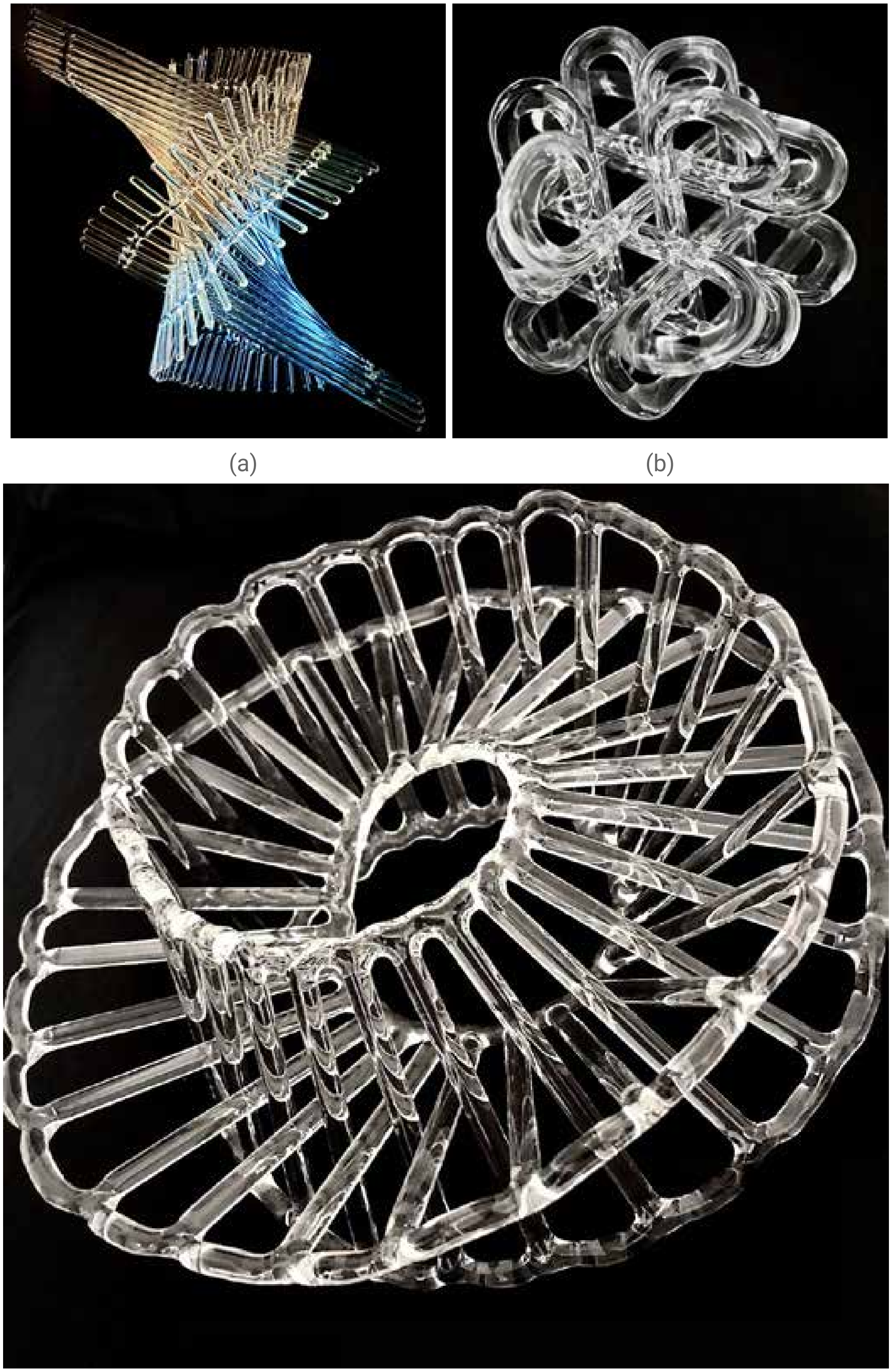
I create sculptures and images that celebrate the intricate patterns that shape our world. Inspired by this underlying geometry, I use borosilicate glass to bring mathematical ideas to life in tangible form. These works range from spirals and knots to interwoven planes, reflecting a variety of abstract concepts. They offer a playful exploration of the intriguing connections and patterns that arise when art and mathematics come together.

Description

My latest work with borosilicate glass explores geometric forms like the helicoid, knots, and Möbius strips. Using flame-working techniques, I manipulate the glass to transform these complex ideas into physical and striking forms. Each sculpture is a blend of precision and playful elements. Clear rods are joined together in symmetric non-intersecting arrangements that invite viewers to reflect on the boundaries between form and emptiness. This work not only presents artistic and technical challenges but also melds scientific inquiry with artistic expression.

Designer(s) Biography:

Anduriel Widmark is a visual artist who blends art and mathematics, transforming complicated ideas into dynamic paintings and glass sculptures. His work features vibrant, abstract forms and detailed geometric structures. He frequently exhibits internationally, including at the Bridges Math Art Conferences, Joint Mathematics Meetings, and the Mathematical Association of America. Anduriel's art explores systems and relationships to develop unique perspectives, celebrating the patterns that shape our world.



(a) (b)

(c)

Figure 1: *Glass sculpture. (a) ‘Tangent Helicoid,’ flameworked borosilicate glass, 18x10x10in. 2024, (b) ‘Hemistix 24 Knot,’ flameworked borosilicate polystix glass knot, 6x6x6 in. 2024, (c) ‘Möbius Strip Quarter Twist,’ flameworked borosilicate glass, 12x12x6 in. 2022.*

Derivatives of Tengstrand’s “3-2-1”

Carlo Sequin
Email: sequin@berkeley.edu
Website: Homepage for Prof. C.H. Sequin
Date Created: May 18,2024
Dimensions: 20cm tall
Materials Used: 3D-print, ABS plastic

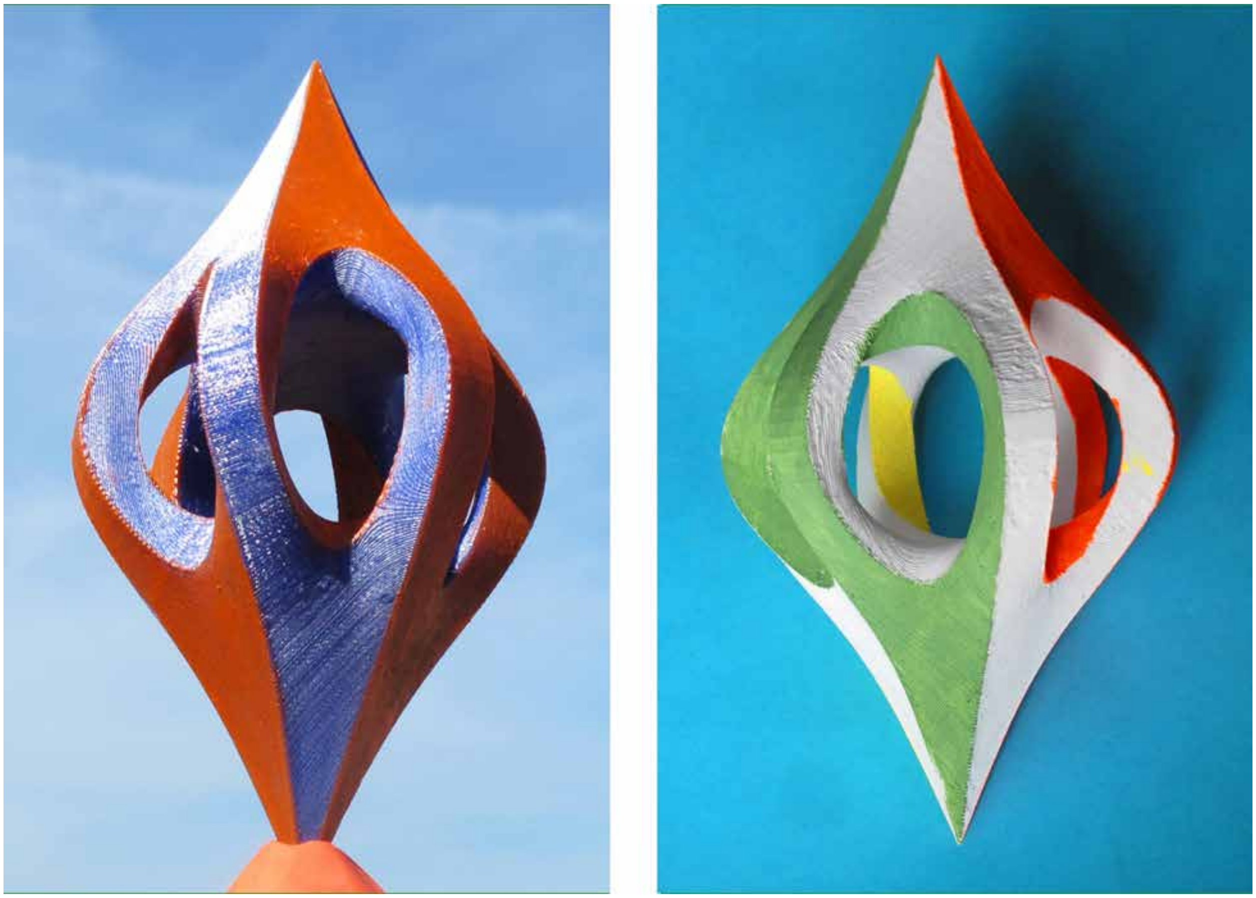
Statement

For several decades I have tried to turn mathematical models into small geometrical sculptures. Often, I start with a particularly intriguing sculpture by a recognized artist. For this exhibition I have been inspired by Tord Tengstrand’s “3-2-1”-sculpture presented at Bridges 2020 [1]. His small model gets its name from the fact that it has three edges, two vertices, but only one single, wildly branching-out, smoothly connected face, that borders itself across the three edges. His sculpture is a handlebody of genus-2 that has three branches in the form of twisted 3-sided prisms. The three ribbon-like surface areas join in two junction patches inside the bi-pyramid structure. In my recent work I explored the structures that arise when I compose more than three prismatic branches and give those prisms more than just three sides.

Description

A particularly interesting case arises when I assemble six 4-sided prismatic branches into a bi-pyramid structure. Here, each individual edge-curves starts at one pyramid tip, makes four “down-up-down-up” passes past the central void, and then ends up at the same vertex that it started from. These six sharp edges partition the surface into six ribbon-like areas that start at one of the inner junction areas, also make four “down-up-down-up” passes, and then merge again in the same junction area, where they join with two other ribbon structures.

Depending on how the three edges starting from the top vertex are oriented with respect to the three edges starting from the bottom vertex, the resulting genus-5 handlebody may have just two “wild” Tengstrand “faces” when three of the six ribbons merge in the upper junction area, and the other three ribbons join in the lower junction patch. But, if one set of three edges is rotated by 60 degrees, one set of three ribbons merges in both junction areas; this prevents the other three ribbons from joining, and this sculpture ends up with a total four Tengstrand “faces.”



“6-2-2” versus “6-2-4” Tengstrand Derivatives”   12cm x 25cm x 22cm tall

An interesting case arises when I assemble six 4-sided prismatic branches into a bi-pyramid structure. Each individual edge-curves makes four “down-up-down-up” passes past the central void and ends up at the same vertex that it started from. These six sharp edges partition the surface into six ribbon-like areas. Depending on how the three edges starting from the top vertex are oriented with respect to the three edges starting from the bottom vertex, the resulting genus-5 handlebody may have just two “wild” Tengstrand “faces” when three of the six ribbons merge in the upper junction area, and the other three ribbons join in the lower junction patch. But, if one set of three edges is rotated by 60° degrees, one set of three ribbons merges in both junction areas; this prevents the other three ribbons from joining, and this sculpture ends up with a total four Tengstrand “faces.”

Designer(s) Biography:

Carlo H. Sequin has been on the faculty of the EECS department at the University of California, Berkeley since 1977. He has been teaching computer science courses with a focus on computer graphics and computer-aided design.

3-fold ‘trefoil knot ‘

Charles Cai

Email: charlescaiysh@gmail.com
Website: charlescai.com
Date Created:2024.6.1
Dimensions:3
Materials Used:N/A

Description

The classical, 3-fold symmetrical trefoil knot is extended and deformed in such a way that the new segments of the knotted strand are hidden behind the visible trefoil image. In this particular design, the extended strand is an unknot resembling the intriguing, ancient Germanic Valknut symbol. This design challenge required the use of a special parallel projection and a knot representation in which all geometric extensions are performed as movements solely in the third dimension, thus remaining hidden behind the original trefoil shape. First, I explored possible desirable extensions in a stick-like, polyline geometry. Later, I then turned this extended knot curve into a cubic B-spline. The two figures show the same 3D shape from the “front” and from a side angle which assumes the intriguing “Trefoil Starman” shape.

Statement

This experiment was inspired by Prof. Henry Segerman’s ingenious presentation of a trefoil knot that disguises itself as a figure-8-shaped unknot when viewed from just the right angle. Last year I embarked on a journey to explore similar knot extensions, gaining invaluable insights and a burst of inspiration. This year, challenged by a suggestion by Prof. Carlo Séquin, I pondered a fascinating twist: What if we could reverse the concept? Let’s start with a classic trefoil from the “front” view, and then extend that curve into some other intriguing shape. This is how I arrived at the “Trefoil Starman,” which is just a specially designed unknot, concealed behind the symmetrical curves of the original trefoil.

On a philosophical level, this artwork may challenge our perception of reality: Is the world truly as it appears to our eyes, or, from some other angles, are there many intricate forms and beautiful new geometries waiting to be discovered? My artwork invites you to look beyond the “surface” and explore the hidden beauty that lies behind it.

Designer(s) Biography:

Charles Yushi Cai is a senior Computer Science student at ShanghaiTech University, currently on leave from UC Berkeley. He recently completed a research internship at the Hasso Plattner Institute in Berlin, focusing on sustainable laser cutting through 3D model adjustments. Charles is also a member of UC Berkeley’s JIPCAD team, working as both a 3D modeling engineer and developer. His research interests center on parametric 3D geometry, with applications in digital fabrication and HCI. Outside of academia, Charles enjoys traveling, photography, and is a passionate Boston Celtics fan!

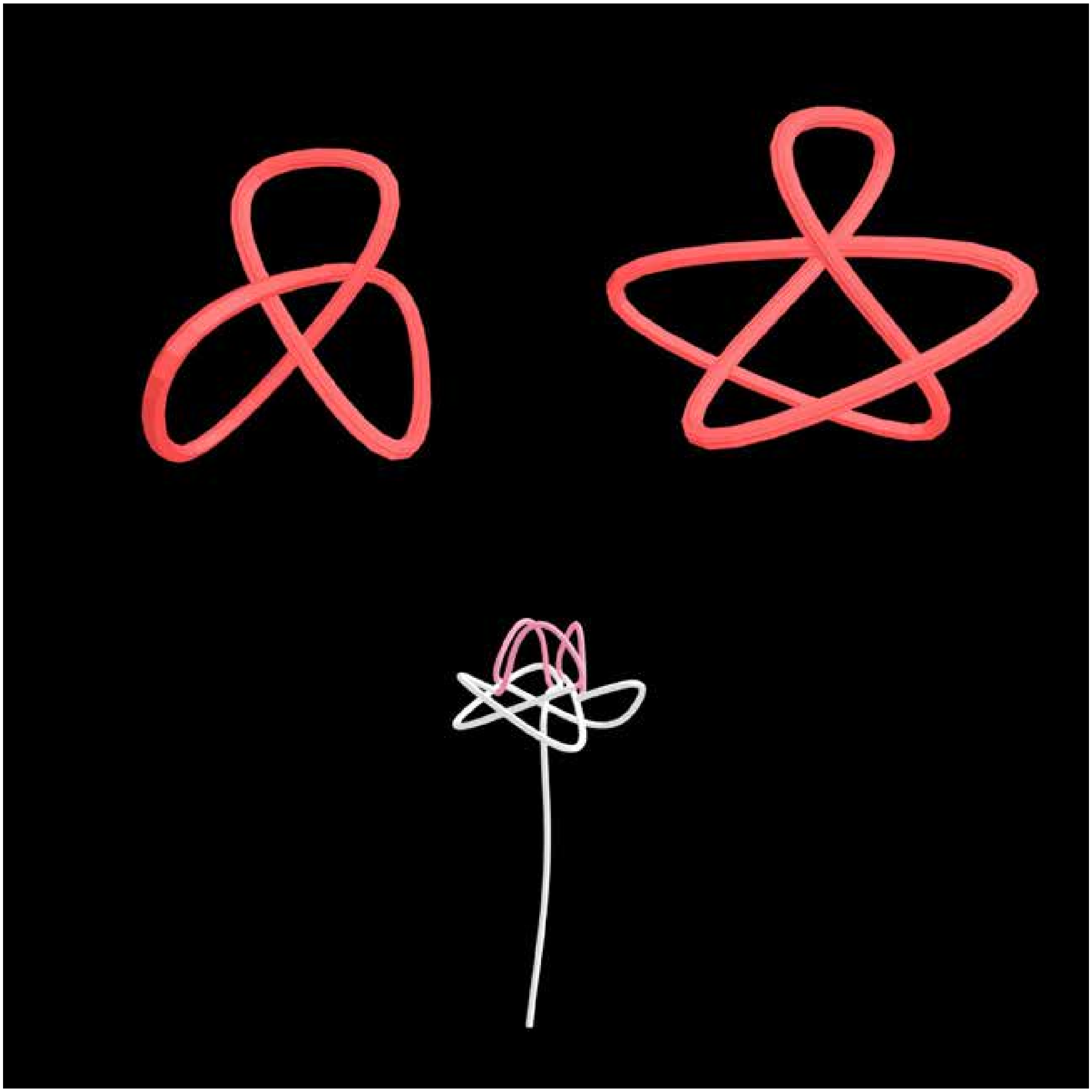


Figure Description: The 3-fold ‘trefoil knot ‘, with a side view of a starman, and a trefoil knot flower based on the 3-fold ‘trefoil knot’ design.

Noble Woman and Hercules

Davide Prete

Email: davide.prete@udc.edu / davide@davideprete.com
Website: https://www.davideprete.com/
Date Created:2023
Dimensions:8x8x14
Materials Used:3D Printed ABS on an alabaster base

Description

The Noble Woman is inspired by traditional Roman sculpture and uses a lattice structure to balance the internal and external space of the sculpture. It investigates the boundary between the surface and what supports the sculptural surface. The investigation started with exploring how space was used for the lost wax casting in bronze sculptures and how new technology, such as 3D printing and lattice structure modeling, can be used as an aesthetic part of the sculpture. The broken areas of the sculpture that show the internal structure are carefully crafted to follow the main lines of the face. A sculpture from the same series, Hercules is a sculpture that uses an innovative lattice structure to create a dialogue between the internal space and the external form of the sculpture. The TPMS used for the module will interact well with the openings of the bust. An alabaster base supports the sculpture.

Statement

The sculptures Noble Woman and Hercules are the results of my latest research investigating the response that lattice structures (that mimic natural patterns, such as honeycombs, leaf veins, or crystal formations as natural forms with an inherent beauty) have with many people.
The sculptures started from a 3D model created by combining 3D scanning and “like clay 3D modeling.” Openings and cuts mimic missing pieces and parts usually present in classical Greek and Roman sculptures.
Traditionally, an unfinished artwork was considered aesthetically and philosophically flawed. However, as we transitioned into the mid-19th century, viewers began to appreciate the allure of incomplete archeological findings. This shift in perspective opened up new possibilities, allowing me to envision the artwork's potential by filling in the missing parts.
By harnessing the power of cutting-edge software (nTopology, NetFabb, Fusion 360, etc.) and employing specific 3D modeling techniques, I embarked on a journey to explore the aesthetic response of adding lattice structures to the sculpture's openings, thereby manipulating transparency, light play, complexity, and intricacy.
The 3D-printed sculptures are finished with an alabaster base.

Designer(s) Biography:

Davide Prete is an Assistant Professor of Art at the University of District of Columbia’s Division of Arts and Humanities in the College of Arts and Sciences. Born in Treviso, Italy, Davide was introduced to the art of metalsmithing by his father, Alessandro, and by the famous sculptor Toni Benetton. He studied jewelry and metalsmithing at the Institute of Art in Venice. In 2003, he obtained his degree in architecture at IUAV, Venice, Italy. Davide has worked as an architect for several architectural firms. He moved to the

U.S. in 2007, and in 2010, he earned his Master of Fine Arts in Sculpture from Fontbonne University in St. Louis, where he studied with Hank Knickmeyer and developed a personal sculptural process, mixing traditional metal casting and new technologies. Later, he specialized in digital fabrication with a certificate from FabAcademy and in Additive Manufacturing with a certificate from MIT.
His work has been shown at national and international venues (Italy, Germany, Czech Republic, England, France, and the USA). His urban-scale sculptures are installed in Italy and the USA.
Recently, his work has focused on new technologies such as 3D printing, laser scanning, AI, and traditional metalsmithing techniques. For his last research, mathematical equations gave him the pretext to connect symbolic images with a new language to discover what he called “a new form of shamanism.”

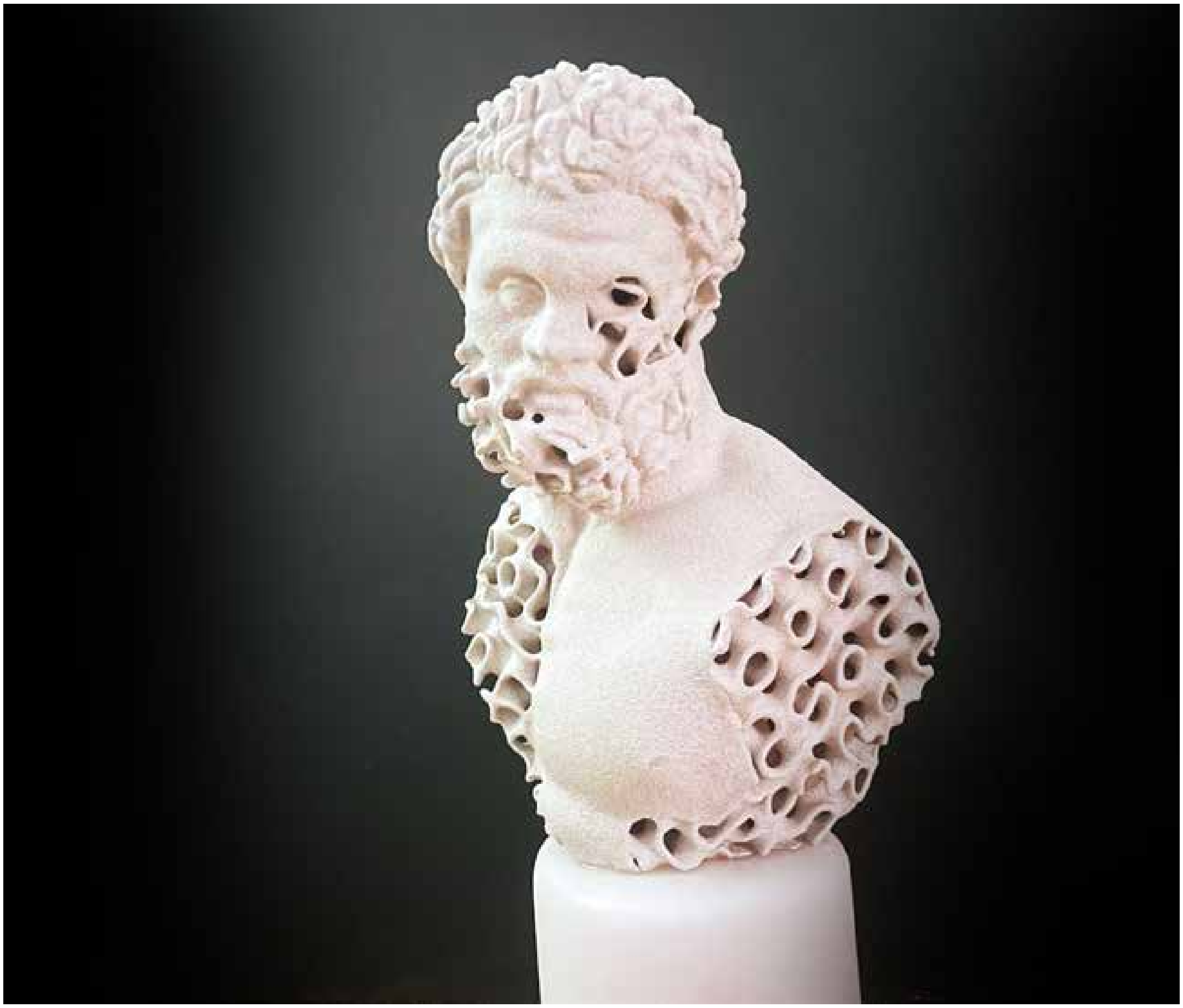


Figure Description: Hercules

MAGNUS POPKO!

Dick Esterle

Email: dickesterle@gmail.com
Website: www.dickesterle.com
Date Created: completed July 2023
Dimensions: 20 inches Diameter
Materials Used: 3D printed PLA model

Description

MAGNUS POPKO!
ASSEMBLED 3D PRINTED MODEL
A sculpture inspired by the work of Ed Popko, Magnus Wenninger and Buckminster Fuller, Kenneth Snelson

After meeting with Ed Popko who forwarded the paper model 60 edge bonded icosahedron model by Magnus Wenninger..
Each icosahedron is replaced with an augmented RB Fuller's Six Part Push Pull Tensegrity, ca. 1979.
The 3 pairs of struts are colored red, yellow and blue which orient orthogonally in an x,y,x manner 90 degrees to each other. They are all identical. There are 8 triangular pieces (the tensional components), 4 black and four white alternating due to chirality of their positions. As a result of a somewhat other than "ideal" model where the edges would occupy the same space, the separation of edges reveals the black white dual which occurs naturally due to the geometry of the configuration.
Furthermore the red yellow blue are oriented such that the "blue" pairs butt ends such that they reveal a triangular and pentagonal patterning which do not coincide to the same surface. The yellow struts always butts with adjacent yellow that form "V" s. The red struts meet 3 to a center and or float individually.

This model involves an ongoing look at these structures as wells suggesting other assemblies as in my talk of acrylic tube and ss wire which produced a 40" diameter structure. As well, 6 axes "geometry machine" models could replace the paper icosahedrons of Wenninger.
See https://vimeo.com/user24186218/3-6axesgeometrymachines

Statement

This piece is more of an investigation and discovery than an illustration. Relationships appeared unexpectedly.

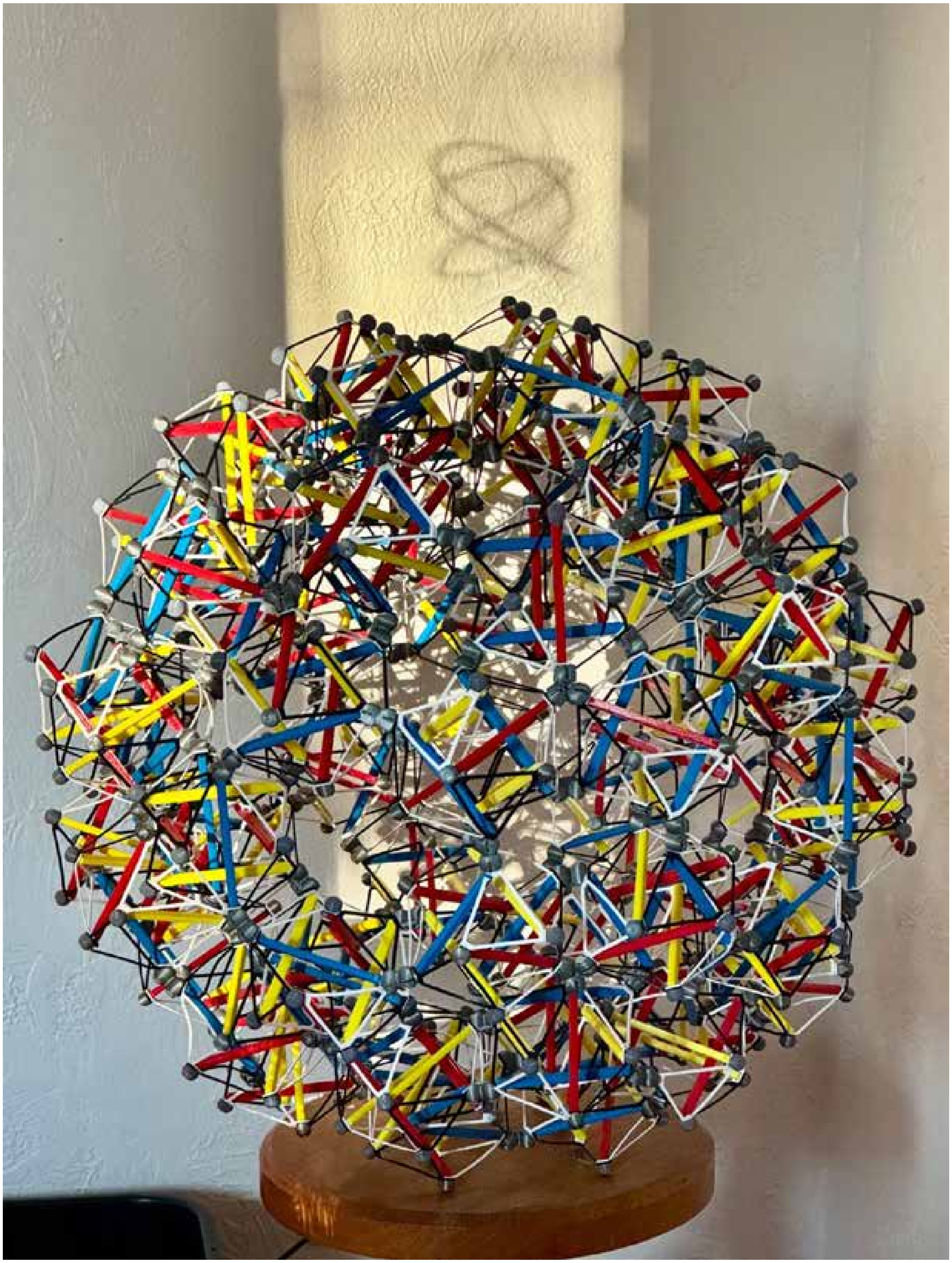


Figure Description: One view of Magnus Popko! with shadow of Bill Gosper's Ambiguous Roller

Stair +

Farah Kaymouz

Email: fkaymouz@cca.edu
Website:
Date Created: 02/26/2023
Dimensions: 14”x12”x16”
Materials Used: wood, canvas fabric, adhesive spray, magnets

Description

Rise + Run is a transformable stair design project by Farah Kaymouz and Jerry Huang for the Materiality & Space 4: TranSTUDIO 2024 Adaptive Design course at California College of the Arts. This project reimagines stairs as dynamic architectural elements that integrate form, function, and sustainability in a seamless transformation. The design features two diamond-shaped components connected by piano hinges, allowing them to transform into a three-dimensional staircase in two different axis. When not in use, one segment ascends, secured by magnets, while the other retracts into the wall, optimizing space and enhancing functionality. This folding mechanism also incorporates a vertical garden, enriching the environment with greenery and promoting ecological harmony. "Rise and Run" challenges conventional stair design by introducing adaptability and multifunctionality into architectural spaces. Through precise iteration and innovative craftsmanship, the project demonstrates how architectural elements can transcend utility to foster aesthetic and sustainable solutions.

Statement

"Rise and Run" explores the potential of architectural elements to adapt and transform, offering a fresh perspective on how stairs interact with space. By merging utility, innovation, and sustainability, the design transforms static stairs into dynamic, multifunctional features. Its dual-fold mechanism introduces a playful experience while maximizing space efficiency, It doubles with an unexpected integration of a green wall feature when not in function, fostering a connection to nature. Through its transformative approach, "Rise and Run" serves as a testament to the evolving possibilities of adaptive architecture while challenging the convention of mechanics.

Designer(s) Biography:

Farah Kaymouz is an interior design student and artist whose work captures the essence of nostalgia through a blend of art and design. With a focus on craft-based studies and material exploration, her process embraces iteration and fabrication to create spaces that evoke memory and connection. Balancing the roles of artist and designer, Farah adapts her approach to each project, shifting between expressive artistry and functional design to achieve synergy.
Jiayue Huang is a senior interior design student passionate about creating beautiful, comfortable spaces that resonate with people. Versatile and imaginative, Jiayue enjoys exploring various materials and fabrication methods for building physical models, along with digital rendering. With a commitment to merging aesthetics and functionality, Jiayue’s goal is to create spaces that are accessible and enjoyable for everyone, enhancing everyday experiences through thoughtful design.

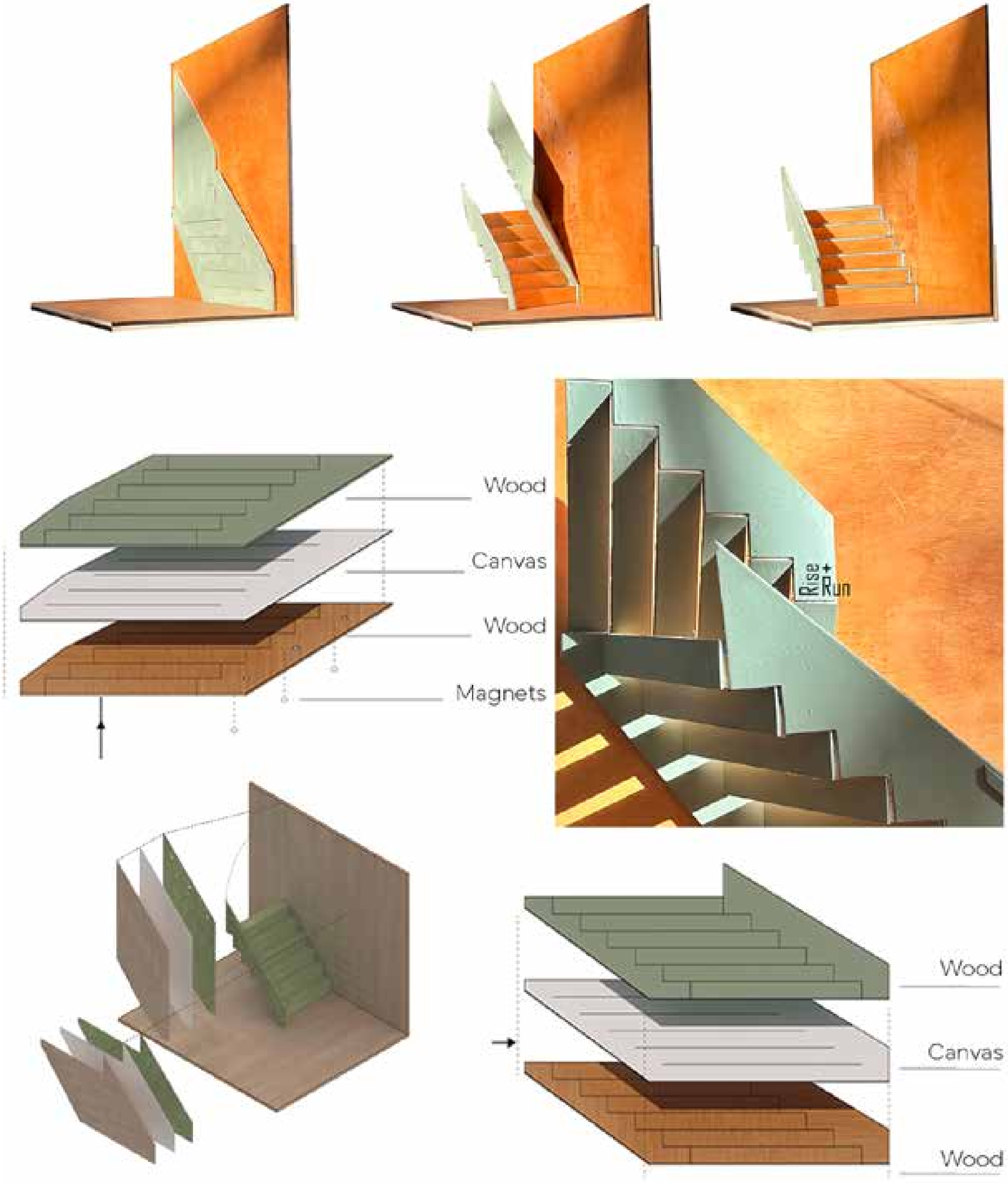


Figure Description: From top to bottom - Physical Model Transformation Stages. Half Transformed Close-up of Physical Model. Digital Diagram of the Components Order and Both Transformations' Axis.

Diatoma

Fatemeh (Shabnam) Lotfian

Email: fatemehsadat.lotfian@hdr.qut.edu.au
Website: https://www.linkedin.com/in/shabnam-lotfian/
Date Created: 2024
Dimensions: 1800*1500*1100 mm
Materials Used: Aluminium, PLA

Description

The Diatoma research explores the intersection of biomimetic design, lightweight structures, and robotic fabrication by studying optimized natural structures and translating these insights into design parameters. Inspired by diatoms—renowned for their intricate and highly efficient structural geometries—the study translates their unique structural forms into design principles for creating a pavilion. The design process incorporates extensive computational modeling, simulation, and prototyping to ensure the final structure achieves optimal strength and lightweight properties. This approach integrates critical fabrication data directly into the design process, establishing a seamless relationship between form, function, and materiality.

The research aimed to develop a material-efficient, highly customized pavilion. The final structure comprises 78 unique component pieces, each formed using Robotic Incremental Sheet Forming (RISF) technology and connected with 3D-printed PLA joints. Spanning 1.5 meters and weighing only 10 kilograms, the pavilion was fabricated and assembled in just 10 days.

Statement

"Diatoma" is the synergy of nature's intricate designs, the precision of computational design, and the innovation of robotic fabrication. The lightweight artwork invites viewers to explore the interplay between natural forms and robotic fabrication, a delicate balance where each component contributes to the strength and resilience of the whole. By gradually deforming the metal sheet, robot not only creates complex shapes but also creates unique textures, curves, and contours. "Diatoma" challenges the boundaries of architectural design, proposing a future where the wisdom of nature is translated through the lens of technology into a tangible, inspiring reality.

Designer(s) Biography:

Shabnam Lotfian is a robotic architect and a PhD candidate at Queensland University of Technology (QUT) and the Building 4.0 Cooperative Research Centre (CRC). Her PhD research focuses on robotic incremental sheet forming for bio-inspired lightweight structures, emphasizing the integration of design and fabrication into a cohesive approach. By utilizing robotic technologies, she develops material-efficient freeform structures. She has showcased her work at notable international exhibitions, including the *International Symposium on Electronic Art (ISEA) 2024* at Arm Hub in Brisbane, *SHErobots 2.0: Ecologies of Care* at Delft University, and *SHErobots 1.0: Tool, Toy, Companion* (2023) at the University of Sydney.

Figure Description: “Diatoma” structure, RISF Process, Component details, Joint & Component Configuration

Surfaces That Tie Photons in Knots

James Mallos

Email: jbmallos@gmail.com
Website: weaveanything.blogspot.com
Date Created: 2024
Dimensions: 12” wide, 10” deep, 7” high
Materials Used: Cardstock paper cut on a Cricut Maker 3

Description

Basketmakers have always been working with smart materials. A thin, straight, flexible strip automatically solves for a smooth surface on which its centerline is a geodesic path.This natural computation becomes even more remarkable if we require that each strand closes smoothly on itself to form a loop, and second, that the basket consists of a single loop. Such a basket corresponds to a knot projection on the sphere. Because closed geodesics are rare, we will need to perturb the basket’s surface with small-scale bumpiness in order to engineer a suitable geodesic.

In 1960, Kunio Murasugi identified a class of knot projections he dubbed *special.* Orienting a special knot projection organizes the entire surface of the sphere into regions of consistent orientation (some clockwise, some counterclockwise) and regions of reversing orientation (where the orientation reverses at each crossing.) Provided there are no reversing regions of size 2, this is tantamount to a topography where clockwise/counterclockwise maps to hill/dale, and reversing maps to saddle.

Statement

By controlling the angle at each crossing, we can force a basket to have weave openings that are hill-like or dale-like (the angles at the corners are made too obtuse to lie flat,) or saddle-like (the angles are made too acute to lie flat.) This paper basket has interlocking notches that force a topography consistent with the special knot projection it embodies. In fact, the required pattern is just a simple alternation (acute on the left, acute on the right, etc.) so a long-enough paper tape notched like that can realize any special knot projection as a basket.

Designer(s) Biography:

I grew up making things out of wood and electronics, but I was late hearing the language of shape. The sculptor in me came out—via the encouragement of others—after my education in engineering had led to designing kites. Most of my work is related to weaving in one way or another. Sometimes a shape just catches my eye and I try to scale it up, but often I am designing woven-work by computer to achieve a desired shape. More and more, the mathematics of how identical pieces can go together has interested me and motivated my work.

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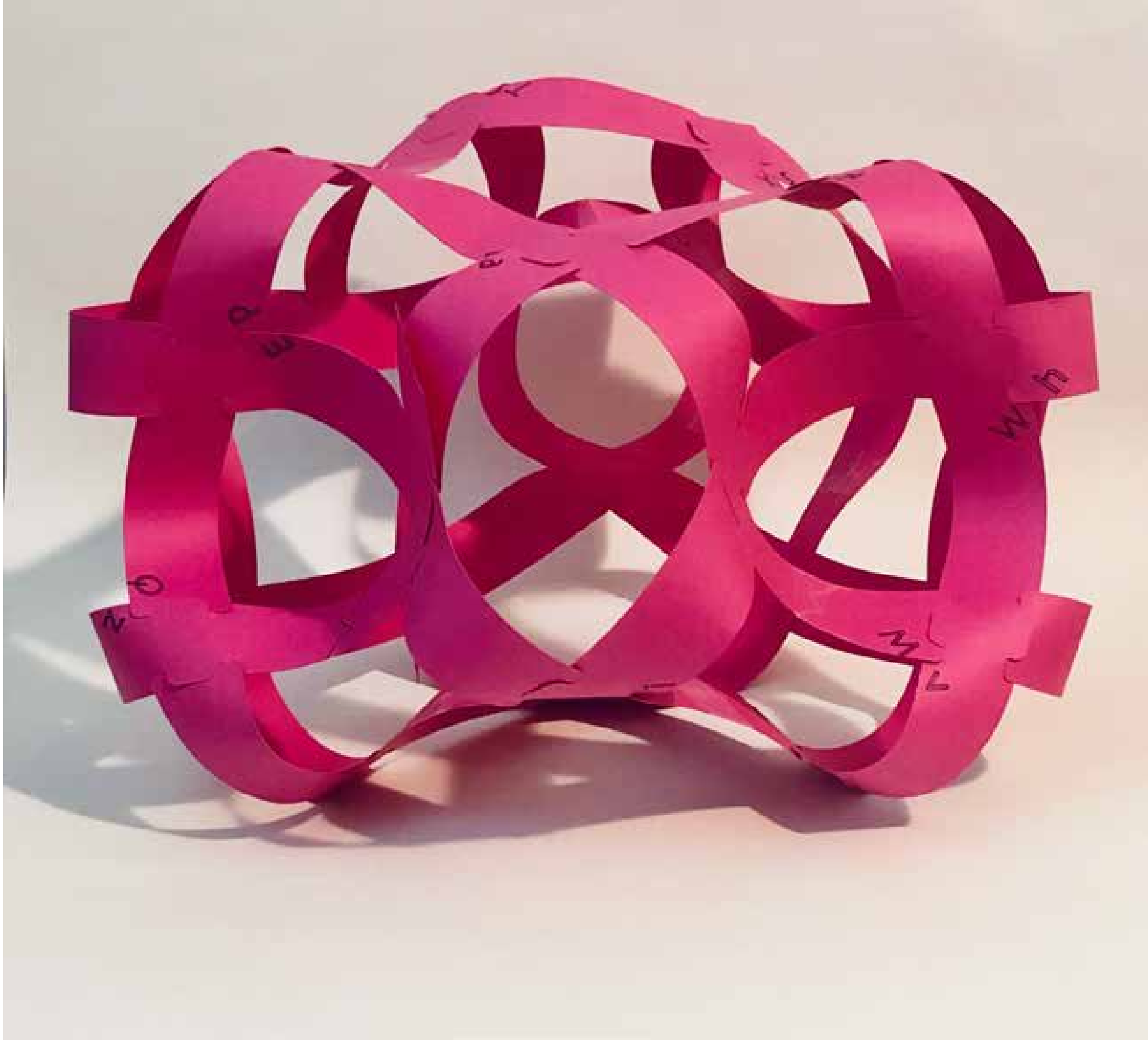


Figure Description: A paper basket corresponding to a single closed geodesic.

Reimagine: Repurposing Digital Fabrication Scrap Materials for Sculpting

Ladan Johari

Email: ladan.joharizadeh@gmail.com
Website: https://www.ladanjohari.com
Date Created: February 2022
Dimensions: 10", 10", 10"
Materials Used: Plywood

Description

Reimagining explores the creative potential of discarded materials, focusing on transforming random plywood scraps into cohesive three-dimensional forms. First, the pieces are systematically collected, and once a taxonomy of materials is established, they are assessed for their aesthetic qualities. The materials are also examined for innovative assembly techniques and structural design to create stable and abstract sculptures. A key challenge of this project was ensuring structural integrity while sculpting with irregular shapes, requiring a balance between material properties and artistic vision. With limited freedom in choosing forms or sizes, the project offers a unique perspective on sculptural practices, encouraging experimentation with imperfect materials and reimagining waste into new and unexpected shapes.

Statement

My work seeks to uncover the creative potential inherent in waste. By employing assembly techniques, I invite a deeper dialogue with materiality, exploring the tension between form and function. Rejecting the use of new materials, this project reflects on the transformative power of discarded fragments, elevating them from the mundane to the extraordinary. It is an exploration of rebirth and how disregarded pieces can be reimagined and granted a second life. The work challenges the sculptural practice, emphasizing the poetic and conceptual value of repurposing, and advocating for a mindful and resource awareness approach to artistic creation.

Designer(s) Biography:

Ladan is an interdisciplinary artist and architect whose expertise in art and architecture naturally draws her to the realm of design. Her work explores the relationship between humans and spaces. As an architect, she has an understanding of the function of spaces, as well as materials, and an eye for the aesthetics of forms and shapes. She began her work as a sculptor in 2017 and has since exhibited her pieces in several exhibitions. Ladan utilizes a variety of materials, tools, and techniques in her work, including bronze, ceramics, and other mixed media.

Figure Description: Left: Structure, Right: Cut pieces

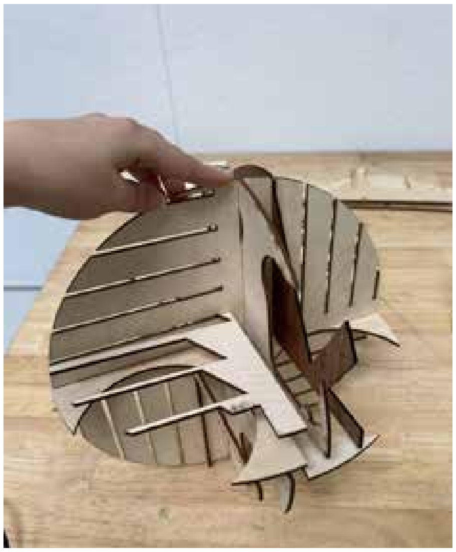


Figure Description: Assembly

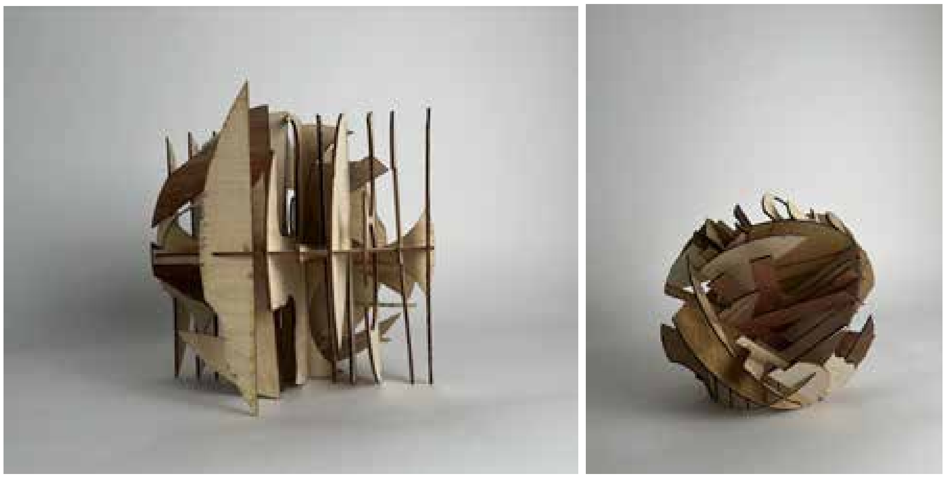


Figure Description: Sculpture pieces, Left: Standalone, Right: Sphere

Fabricated Combines

Sara Codarin & Masataka Yoshikawa

Email: scodarin@ltu.edu, myoshikaw@ltu.edu
Website(s): www.saracodarin.com www.ltu.edu/content/faculty-and-staff/masataka-yoshikawa
Date Created: June 2024
Dimensions: 8.5”x8”x8”
Materials Used: 3D Printed Resin

Description

The design research, Fabricated Combines, explores the integration of generative AI into architectural design, investigating the interplay between aesthetics and architectural constructs. Drawing inspiration from Robert Rauschenberg's concept of "combine," the project reimagines AI-generated two-dimensional images as starting points for creating innovative spatial and material designs. By blending generative AI-driven approaches and traditional design processes, the study uses platforms for image generation, 3D modeling, and digital sculpting to translate AI visuals into three-dimensional geometries. The iterative workflow bridges digital and physical realms, incorporating 3D printing and manual manipulation to refine and inform designs. This exploration challenges conventional notions of space and materiality while showcasing AI's potential to accelerate and enhance creative production. While AI serves as a powerful tool for synthesizing ideas, human designers remain integral to refining outputs, ensuring innovation is grounded in human creativity and authorship.

Statement

Our design philosophy embraces the transformative potential of AI-integrated workflows to push the boundaries of architectural exploration. By leveraging generative AI as a creative partner, we aim to transcend conventional design methodologies and their workflow. Through iterative processes that blend AI-generated visuals, procedural modeling, and physical fabrication, we challenge traditional notions of space, materiality, and scale. AI serves not as a replacement but as a catalyst for innovation, enabling a continuous feedback loop between the digital and the physical. This synergy between human creativity and AI-driven synthesis redefines design possibilities, fostering a new era of exploration and transformative architectural expression.

Designer(s) Biography:

Sara Codarin and Masataka Yoshikawa are assistant professors of architecture at Lawrence Technological University, College of Architecture and Design, in Southfield, Michigan. Sara, a techno-optimist with a Ph.D. from the University of Ferrara, specializes in robotic manufacturing for Cultural Heritage conservation. Her work, tied to economic, technological, and social shifts, spans robotic fabrication, digital craft, ecological issues, and generative AI for storytelling and world-building. Masataka explores how architects adapt design workflows in the digital era. His research integrates 3D simulations, virtual/physical models, VR, MR, and digital fabrication. Together, they shape the future of architecture through technology-driven innovation and teaching excellence.

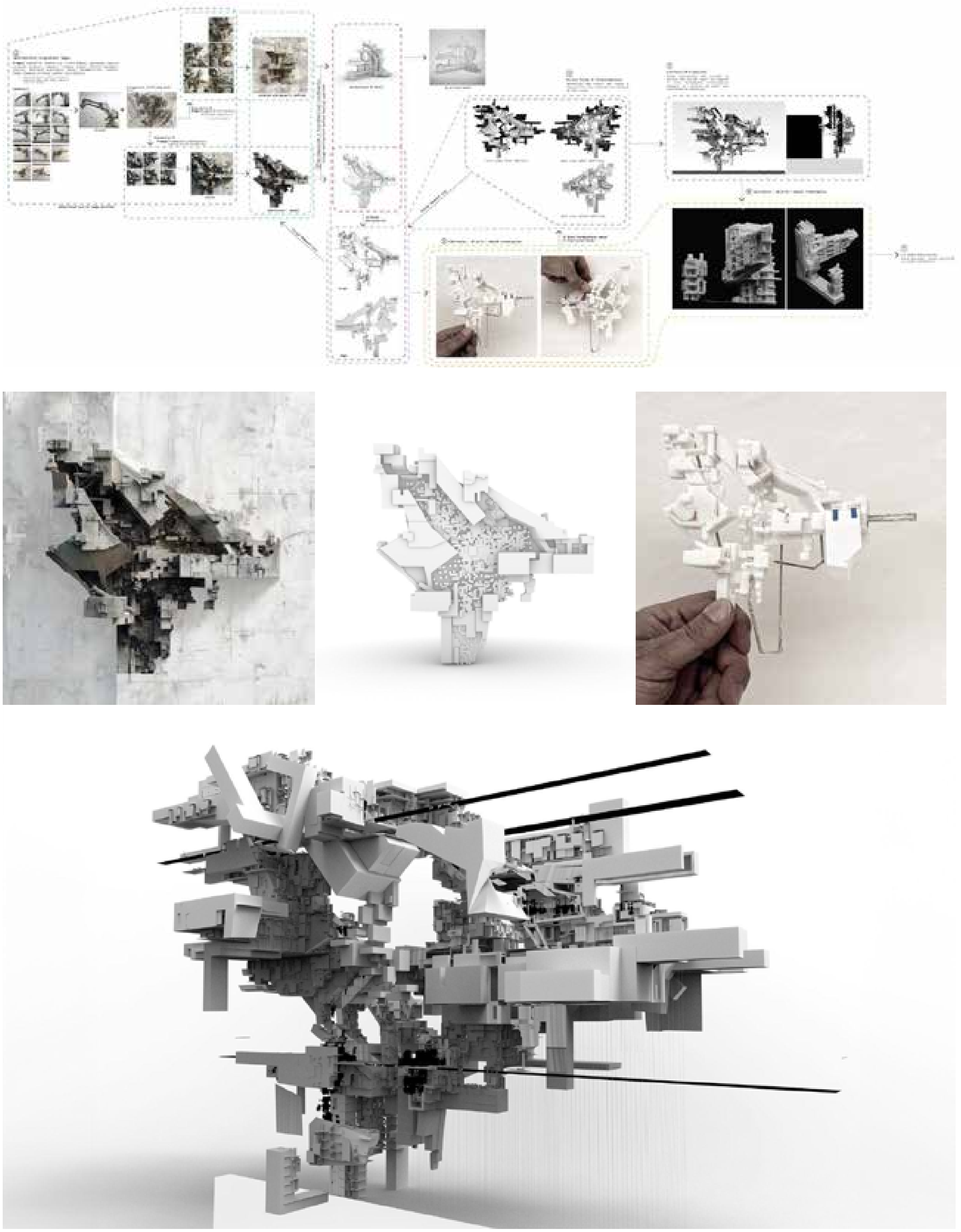


Figure Description: AI-driven workflows and 3D printing converge as an atlas of architectural exploration, redefining spatial and material creativity through innovative processes.

Cavilion (DigiPy)

Mohammed Behjoo

Email: behjoo.mohammed@gmail.com

Website: architizer.com/projects/cavilion/
Date Created: August 10, 2023
Dimensions: 5 m × 5 m
Materials Used: Plywood

Description

The DigiPy workshop was a hybrid digital fabrication workshop organized by DigitalFUTURES in collaboration with Dahi Studio and Digital Craft House. This workshop aimed to familiarize participants (48 students) with computational design and digital fabrication, providing them with the opportunity to construct a pavilion on a 1:1 scale. After learning about the computational design and general concepts of digital fabrication, students started designing and proposing various forms for the final pavilion. The best design alternative was selected for proceeding to the design development and shop drawing generation phase. From the production of the structure's elements using a three-axis CNC machine, to the final assembly of the elements, students were actively involved in gaining hands-on experience in the digital fabrication process. The final bending-active structure, named "Cavilion", was assembled at the University of Tehran in three days.

Statement

The essence of the project is based on the utilization of bending to induce elastic deformations using the active-bend method. The rationale for employing this method is predicated on its inherent simplicity in manufacturing planar components, which are subsequently configured into curved elements during the assembly process. Among the form alternatives designed by students, the one that received widespread approval, in terms of aesthetics, stability, and constructability, was selected for development. The form's anticlastic curvature rendered its surface undevelopable, preventing flattening without distortion. To address this, it was segmented into three strips (radial, peripheral, and patches) for stability.

Designer(s) Biography:

Danial Keramat: An experienced computational designer and programmer with an architectural background. Danial has a deep passion for computational geometry and automating BIM-related tasks.

SeyedAli Derazgisou: A Ph.D. student at VT researching AI-driven computational design, architectural automation, and façade engineering. He has extensive experience in teaching, conducting workshops, and collaborating on multidisciplinary projects.

Kaveh Khodabakhshi: A computational designer specializing in design automation and computer-integrated construction. He is interested in pedagogy and actively engages in teaching-based fabrication projects.

Mohammed Behjoo: An AI researcher, and a Python developer with a strong focus on computer vision.

Photo by Sogand Malekloo

Diagram by Nadia Kazemi

Photo by Fatemeh Jebeleh

Figure 1. Three layers of the structure: peripheral, radial, and reinforcement patches

Adaptive 3D-Printed Textiles: Flexibility and Rigidity On Demand

Negar Kalantar + Alireza Borhani

Email: negarkalantar@gmail.com
Website: thetranslab.com
Date Created: 2014
Dimensions: variable
Materials Used: P12

Description

In our artwork, we explore the dynamic relationship between flexibility and rigidity through 3D-printed textiles. Using innovative design techniques, we have created a material that can transform its shape when force is applied and maintain that shape once the force is removed. This unique characteristic allows the textile to be both flexible and rigid, adapting to different forms as needed.

We employ 3D printing technology to fabricate these textiles, using precise geometric designs to achieve the desired properties. The process involves meticulously designing each cell to ensure the assembly holds together through kinematic constraints, enabling the material to return to its original form when reversed forces are applied. Our focus is on how geometry can alter material properties, creating a responsive, adaptable textile from traditionally rigid substances. This work challenges conventional design discourses by merging scientific investigation with creative exploration, offering new possibilities for material innovation.

Statement

As artists and designers, we embrace the opportunities that new tools bring to our creative process. 3D printing, in particular, has revolutionized our approach, allowing us to achieve unprecedented levels of complexity and accuracy in our designs. This technology enables us to create textiles that do not need to be stitched to form three-dimensional structures; instead, they can be printed flat and transformed into 3D shapes upon applying force.

Our work focuses on manipulating rigid materials into flexible, responsive textiles, harnessing the power of geometry to redefine their properties. This innovative approach challenges conventional perceptions of rigidity and flexibility, merging scientific investigation with artistic creativity. Each piece we create is a testament to the potential of design to innovate and inspire, inviting viewers to rethink the possibilities of the materials surrounding us. Our work aims to spark curiosity and encourage a deeper appreciation for the fusion of art and science.

Designer(s) Biography:

Negar Kalantar, Ph.D., is an associate professor of Architecture and the Co-director of the Digital Craft Lab at California College of the Arts in San Francisco. Her research focuses on materials exploration, robotic and additive manufacturing technologies, and the integration of architecture, science, and engineering.

Alireza Borhani is an innovator, architect, educator, and co-principal of transLAB. His interdisciplinary experience has broadened his career across a diverse range of projects at the intersection of design computation, emerging material systems, additive manufacturing workflows, and robotics. Leading in kinematic and lightweight structures, ranging from architectural-scale shelters to small products.

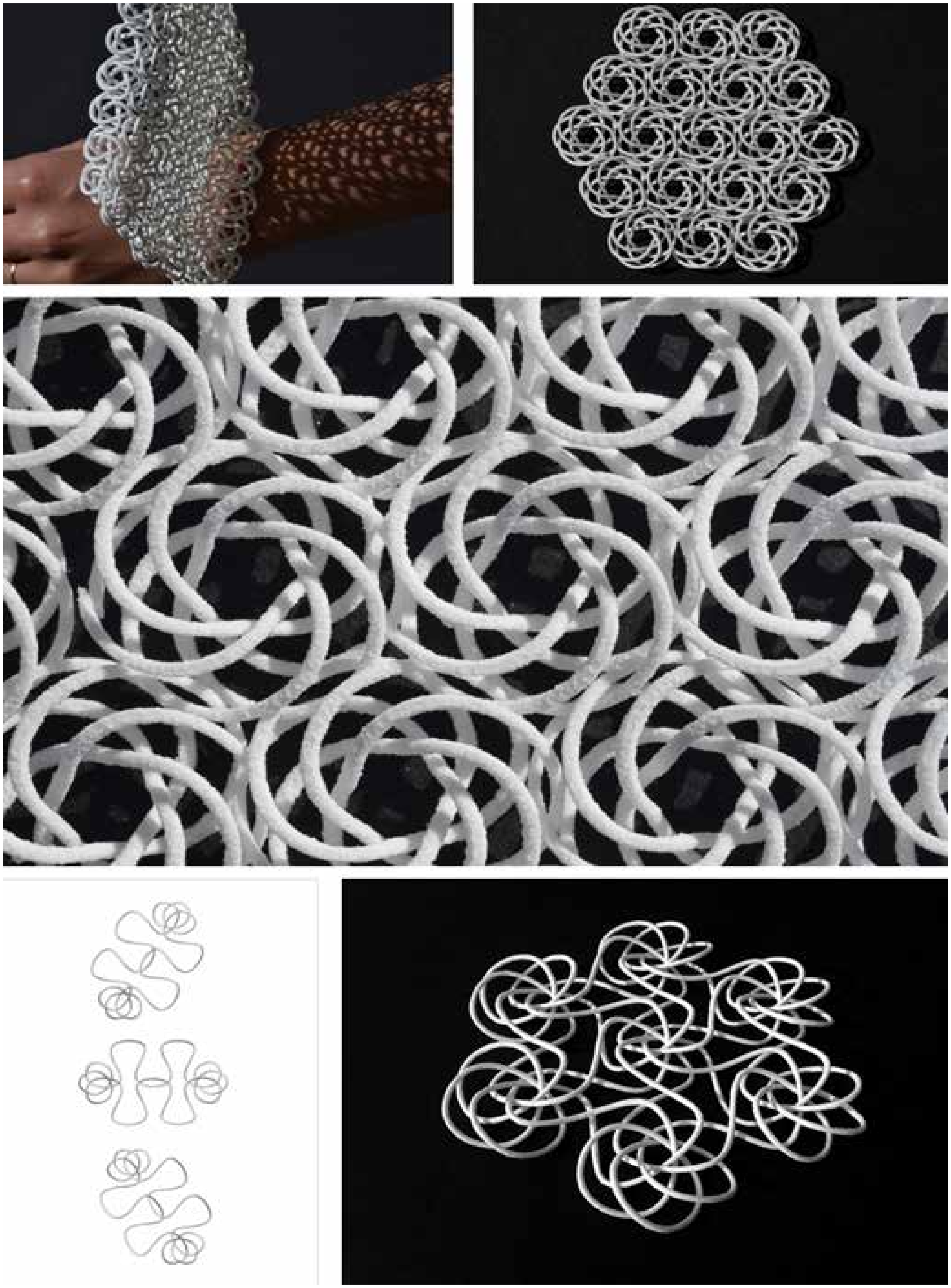


Figure Description: Parametrically designed textiles for 3dprinitng

Galaxy Lamp “Andromeda S”

Phil Webster

Email: phil@philwebsterdesign.com
Website: https://www.philwebsterdesign.com/

Date Created: 2023
Dimensions: 11" x 24" x 11"
Materials Used: Cherry, basswood, washi paper

Description

The Galaxy Lamp series explores a set of shapes called polar zonohedra that feature spirals along their surfaces. By extruding these spirals down from edges of the outer shape, I create a simple-yet-complex dance of multiple spirals starting and ending at two common points. Beautiful in their own right as small tabletop sculptures, the real magic happens when the lamps are turned on. The light fills the inset washi paper in each face with a golden glow, instantly creating a soothing, meditative ambiance in any room.

Each lamp starts with a 3D computer model. The individual parts are cut out of laminate with a basswood core and surface veneer of natural cherry, as well as genuine Japanese washi paper made from mulberry. Each panel is glued together from two wooden frames and a piece of washi paper, and then the entire structure of the lamp is hand assembled and glued around a simple black stand.

Statement

I've had a lifelong love affair with geometry. I take ancient geometric traditions and combine them with modern mathematical concepts (fractals, polyhedra) and technology (3D modeling, laser cutting) to create unique, contemporary art and décor. I capture ancient traditions with a modern twist. All of my work stems from one core impulse: to celebrate the inherent beauty of mathematical forms. I believe we all share an innate appreciation for symmetry and pattern. There is something sacred in the creation and viewing of these forms that allows me to meditate on the infinite patterns present in the deep structure of our world.

Designer(s) Biography:

Phil Webster is a self-taught artist exploring the intersections of mathematical patterns and shapes, natural forms, ancient design traditions, and sacred geometry. He blends his distinctive and diverse set of skills and interests with his lifelong love for the beauty of geometry to produce works of meditative, contemporary art and décor. His studies have been in the fields of geometry, cognitive science, music, cartography, and computer science. The common threads running through these fields inform his art and design today, including the appreciation of technological tools, aesthetics and design skills, and mathematical and programming skills.

Galaxy Lamp “Andromeda S”

Ceramics-based Mixed-media Möbius-band Sculptures

Robert Fathauer

Email: robertfathauer@gmail.com
Website: robertfathauer.com

Date Created: 2023/4
Dimensions: Multiple pieces, each measuring 18-26 cm in all directions
Materials Used: Ceramics with wood, stone, raffia, leather, metal wire, and/or cane

Statement

My work explores the mathematics of symmetry, knots, fractals, tessellations and more, blending it with plant and animal forms as well as inorganic forms found in nature. This synthesis allows me to create innovative prints and sculptures that derive their beauty from a combination of complexity and underlying order. My goal is to use mathematics in my work in a manner that is compelling to those who understand it but does not serve as an impediment to those who don't.

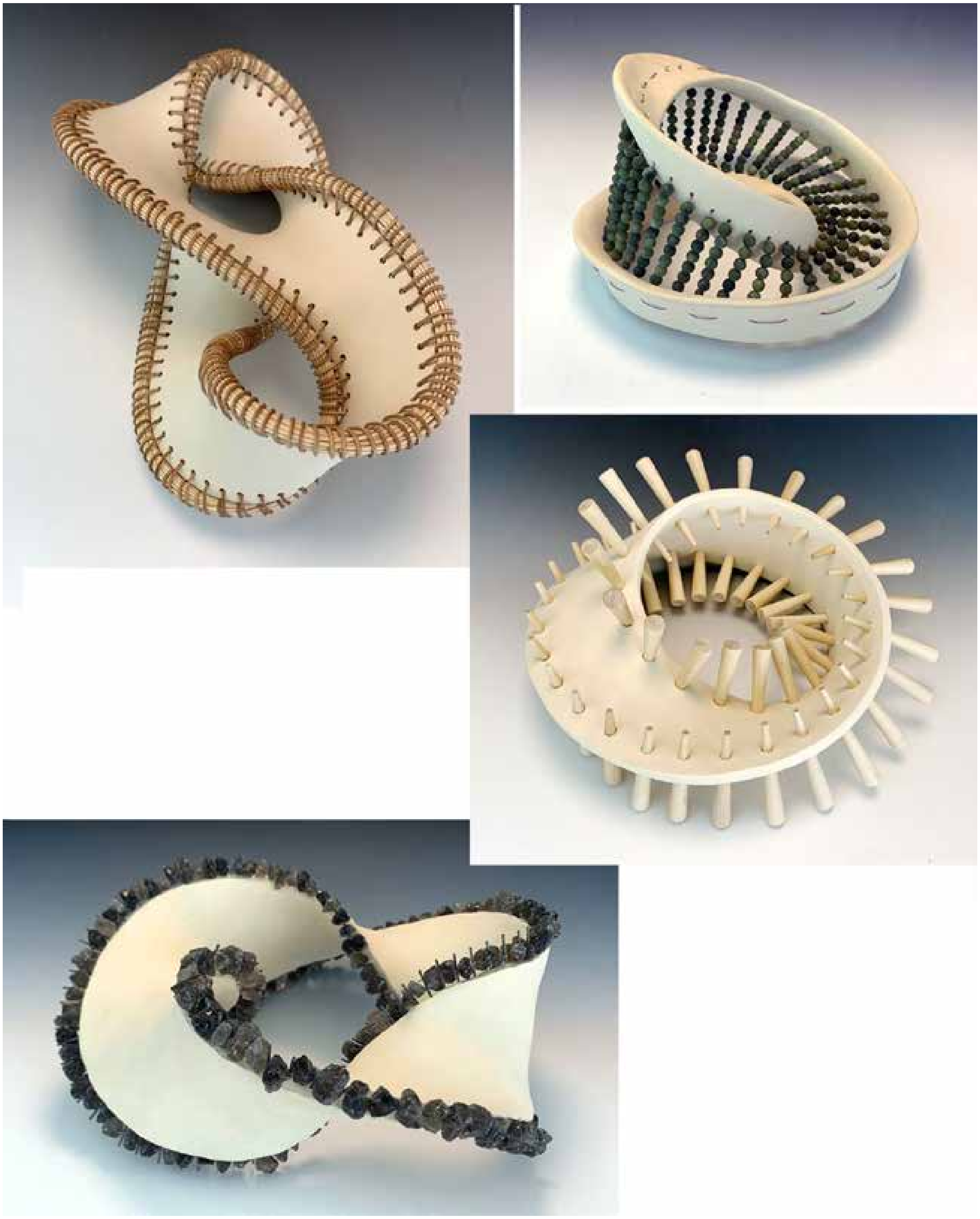
Description

I present a group of four hand-built ceramic topological sculptures that make use of additional materials, including cane, leather, raffia, wood, metal wire, and stone beads. These other materials add uniqueness and interest in addition to delineating aspects of the mathematics underpinning the work. In particular, the different materials distinguish edges from surfaces in pieces based on Möbius strips and on knots with their associated Seifert surfaces.

In the first piece, the ceramic portion is based on the Seifert surface for the figure-8 knot, while the knot is formed with cane strips and affixed to the ceramic part with leather cord. In the second, a bowed ceramic ribbon describes the edge of a Möbius band, while rows of natural jasper beads strung between portions of the ribbon define the band itself.In the third, wooden pegs are used to add interest to a Möbius band and to emphasize the single edge. In the last piece, rough natural smoky quartz beads are woven along metal pegs to decorate the edge of a three-half-twists Möbius band. The line of quartz describes a trefoil knot.

Designer(s) Biography:

Robert Fathauer has a PhD from Cornell University in Electrical Engineering and has been making art incorporating mathematics for over 30 years. He has written numerous papers on his explorations in recreational mathematics and took the lead in making art exhibitions an annual feature of both the Bridges Conference and the Joint Mathematics Meetings. He is the author of the 2021 book “Tessellations: Mathematics, Art, and Recreation”.



Clockwise from upper left: Figure-8 knot formed with cane strips, with ceramic Seifert surface; Möbius band defined by jasper beads supported by a ceramic ribbon; ceramic Möbius band with wooden pegs, and three-half-twists band decorated with rough smoky quartz beads.

topological crochet

Shiying Dong

Email: shiyingdong@gmail.com
Website: https://www.instagram.com/clay_mushi/
Date Created: 03/16/2024
Dimensions: 36.0 x 36.0 x 36.0 cm
Materials Used: Wool, nylon, copper

Description

This piece showcases the power of topological crochet technique. It is in the same series as my exhibit pieces in Bridges 2023 and JMM 2024. In these, I use the same method to build a yarn sculpture that manifests the chiral icosahedral symmetry. This piece is a surface bounded by a link of six near-flat coils that intertwine each other symmetrically. The surface is nonorientable with genus 86. The construction starts from a foundation chain graph made from twelve (2,5) knots lining up like a dodecahedron, with saddle joints appropriately set up, using a method I discovered during topological crochet exploration. This method was taught in a workshop in Bridges 2024.

Statement

Creating with my hands connects me to the world. Through explorations of shape and form, I seek a deeper understanding of our surroundings. I find joy in the tactile sensation of yarn flowing through my fingers, and as if answering to the magic power of mathematics, strands become stitches, and stitches coalesce into structures in a fine way that honors their intrinsic nature.

Designer(s) Biography:

Shiying Dong is a pioneering fiber artist merging mathematics and art through abstract sculpture. With a background in theoretical physics and mathematics, Shiying developed Topological Crochet, a groundbreaking style translating algebraic topology concepts into yarn sculpting.

Shiying teaches workshops at the National Museum of Mathematics and has led sessions at the Bridges conferences. Her work has been showcased at Bridges
and the Joint Mathematical Meetings. Shiying shares her techniques on YouTube and is co-authoring "Unravelling Topological Crochet" with math artist Eve Torrence.

Expanding her creative horizons, Shiying explores laser cutting, earning the prestigious 2023 Einstein Mad Hat Award Grand Prize.

Figure Description: Saddle Monter, a nonorientable surface with genus 86, bounded by a link of six coils that intertwine each other symmetrically.

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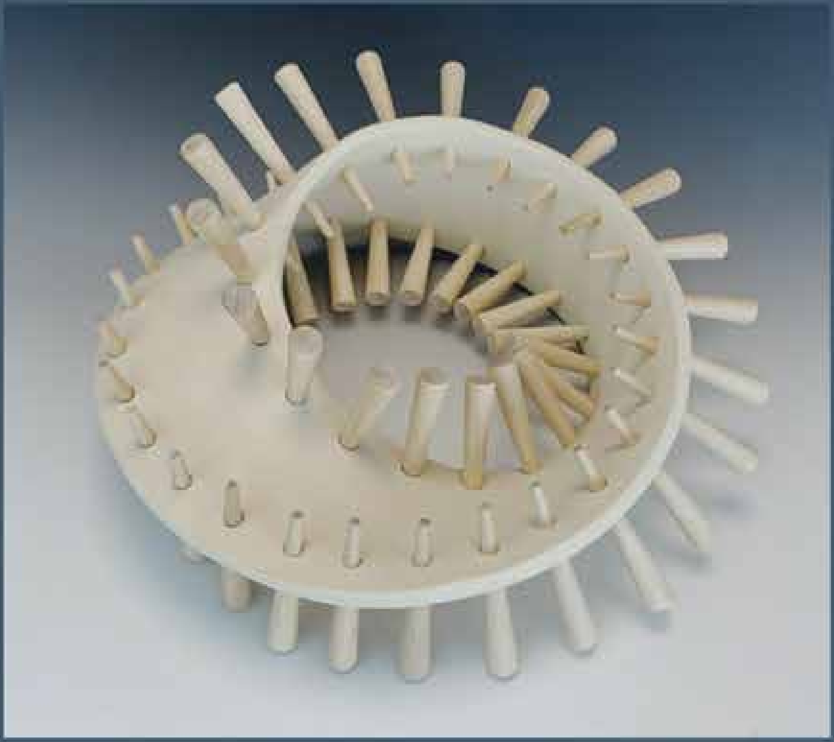

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